Insight inside out:
A comparison between the phenomenon and the experiments designed to study it

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Sander V. Crielaard

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Radboud Universiteit Nijmegen

Begeleider: Prof. dr. Anna M.T. Bosman
Tweede lezer: Drs. Marianne W.L. Reuling

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Abstract

Sometimes after not thinking on a problem the solution to that problem suddenly emerges. This experience is common to most of us and has been noted by various writers. Yet the existence as a real phenomenon and its nature are hard to grasp scientifically. The aim of this paper was to compare three types of approaches to the phenomenon, that is the description of the phenomenon based on personal experiences and two lines of experiments, which investigated (certain aspects of) the phenomenon. The first line contained a distraction task as the central part of the experiment, whereas the second line did not focus on distraction in particular. All three types of approach were evaluated by applying the framework of Wallas (1926/1949) on them. Wallas' framework contains four steps (preparation, incubation, illumination, verification). Furthermore, the measurement principles of Borsboom, Mellenberg and van Heerden (2004) were used as a guideline to examine the validity of the research methods of the experiments. This procedure led to the four conclusions. One although the description of the experiences were often characterised by the four steps of Wallas, the process underlying the phenomenon was not per se a reflection of those steps. Two the research methods in all experiments (except for Stephen, Dixon, and Isenhower, 2009) were inappropriate to capture the process underling the phenomenon. Three there was little consensus about what the phenomenon exactly is and what the appropriate way would be to investigate it. Four, before conducting experiments, there should be a theoretical framework (e.g., catastrophe theory, complex system theory) about what the phenomenon is and how it manifests itself. Based on this knowledge, a measurement should be decided upon that takes individual differences in experiences into account when investigating the process that leads to sudden insight.

> I was sitting writing at my textbook but the work did not progress; my thoughts were elsewhere. I turned my chair to the fire and dozed. Again the atoms were gambolling before my eyes. This time the smaller groups kept modestly in the background. My mental eye, rendered more acute by visions of this kind, could now distinguish larger structures, of manifold conformation; long rows, sometimes more closely fitted together, all twining and twisting in snakelike motion. But look! What was that? One of the snakes had seized hold of its own tail, and the form whirled mockingly before my eyes. As if by a flash of lightning I awoke.
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> Kekulé translated by Benfey (1958, p. 22)[1]

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1 Kekulé, on occasion of the 25th jubilee of the benzene theory, recalling in 1890 one afternoon, that must have been somewhere in the first half of 1862 (Rocke, 1985). The insight, on that afternoon, was the inspiration for his 1865 paper on the benzene ring (Sur la constitution des substances aromatiques. *Bulletin de la Societe Chimique de Paris* 3 (2): 98–110). Benzene was included in a large group of aromatic molecules that shared that they all had a strong odor. Latter research revealed that although the strong odor was a very obvious characteristic, the structure of the molecules in this group actually differed. Thus the group of aromatiques was only apparent. The approach to describe molecules by there valences that determined the number of bindings an atom could have, was discovered some years earlier. The problem for the benzene molecule was that there were more bindings than is should structurally have, yet

> In working mathematical examples in the evening I some- times 'get stuck.' I leave it over night and take it up in the morning and I often get the answer immediately. So in translation I find passages that I cannot get out. I study on them for a while and then leave them for several hours, or better sometimes days, and I can get them clearly.
>
> Anonymous in Child (1892, p. 455)

Insights in the structure of nature like Kekulé had are rare; they occur only incidentally. They appear seldom even to the great minds. Although we have not all had insights like Kekulé, the anonymous quote could, however, have been from any of us, apart from some variations in the kind of problem. Probably most people recognize situations, wherein one thinks about a problem for a long time without getting a solution. After some failing attempts there has to be a break and the problem is left untouched for a while, because one simply has other things to do. Sometimes this break turns out to be beneficial. Some ideas arise at totally unexpected moments, while walking the dog or taking a shower. The experience is noticed by scientists, artist and writers. A couple of collections of such experiences have been reported by amongst others, Carpenter (1875), Child (1892), Ghislin (1954), Hadamard (1945), Koestler (1964), and Wallas (1926/1949)⍰. There have been different names for the process underlying this experience, such as, unconscious cerebration (e.g.,Carpenter, 1875), incubation (e.g., Patrick, 1938; Gall & Mendelsohn, 1967), and lately unconscious thought (e.g., Dijksterhuis, 2004). Note however, that the nature of the underling process of the experience is not at all clear! The experience appears to be hard to catch in an experiment. This let some to conclude that the experience of incubation might be an illusion (Olton, 1979)⍰. Does something exits if it cannot be measured?

The main goal of this paper is to compare three approaches to study the phenomenon, that is, the description of the phenomenon based on personal experiences and two lines of experiments, which investigated (certain aspects of) the phenomenon experimentally. I will use the framework of Wallas (1926/1949) to assess all three types of approaches and the definition recarding validity (operationalization) of Borsboom, Mellenberg and van Heerden (2004) to assess two lines of experimental manipulations. One line of experiments typically provided the participants with a problem, then introduced a distraction task to one and not to the other group, and finally evaluated the effect of the manipulation by the outcome, a performance measure. That is probably the most salient aspect of the phenomenon. The insight follows incubation, that is a period of not thinking of (i.e., being unconscious of or not remembering) the problem. Therefore a quite obvious manipulation is distraction to give the emergence of insight a chance. Accordingly the experiments can be divided into two lines of research, namely, experiments that concentrate on some form of distraction and experiments that do not. The phenomenon and the experiments, that were meant to test them, are compared using Wallas' four-step framework. It is argued that not all experimental set-ups are capable of measuring the phenomenon.

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it did not bind with other molecules such as chlorine and … that could attach to the free valence. This was solved by Kekulé's insight that the benzene molecule bonded to itself, like a snake biting it tale. This explained why the benzene molecule was stable, all valences had a bond.

1 A closer look at the phenomenon of insight

Probably the most famous example is Poincaré's experience of discovering the Fuchsian functions. He devoted an entire paper, *Mathematical creation (Poincaré, 1908/2000)*, to just this experience. He states that it is merely one example of how problem solving generally worked for him. Many experimenters used Poincaré's description as an inspiration for investigating the phenomenon (e.g., Bowden, Jung-Beeman, Fleck, & Kounios, 2005; Dijksterhuis, 2004; Dreistadt, 1969; Orlet, 2008; Patrick, 1938; Smith & Blankenship, 1991; Yaniv & Meyer, 1987). Note that, Poincaré actually recounts not one but four of such experiences (A-D). All four are presented

A:

> For fifteen days I strove to prove that there could not be any functions like those I have since called Fuchsian functions. I was then very ignorant; every day I seated myself at my work table, stayed, an hour or two, tried a great number of combinations and reached no results. One evening, contrary to my custom, I drank black coffee and could not sleep. Ideas rose in crowds; I felt them collide until pairs interlocked, so to speak, making a stable combination. But the next morning I had established the existence of a class of Fuchsian functions, those which come from the hypergeometric series; I had only to write out the results, which took but a few hours.
>
> Then I wanted to represent these functions by a quotient of two series; this idea was perfectly conscious and deliberate, the analogy with elliptic functions guided me. I asked myself what properties these series must have if they existed, and I succeeded without difficulty in forming the series I have called theta-Fuchsian. (p. 88-89)

B:

> Just at this time I left Caen, where I was then living, to go on a geological excursion under the auspices of the school of mines. The changes of travel made me forget my mathematical work. Having reached Coutances, we entered an omnibus to go some place or other. At the moment when I put my foot on the step the idea came to me, without anything in my former thoughts seeming to have paved the way for it, that the transformations I had used to define the Fuchsian functions were identical with those of non-Euclidean geometry. I did not verify the idea; I should not have had time, as, upon taking my seat in the omnibus, I went on with a conversation already commenced, but I felt a perfect certainty. On my return to Caen, for conscience' sake I verified the result at my leisure. (p. 89)

C:

> Then I turned my attention to the study of some arithmetic questions apparently without much success and without a suspicion of any connection with my preceding researches. Disgusted with my failure, I went to spend a few days at the seaside, and thought of something else. One morning, walking on the bluff, the idea came to me, with just the same characteristics of brevity, suddenness and immediate certainty, that the arithmetic transformations of indeterminate ternary quadratic forms were identical with those of non-Euclidean geometry. (p. 89)

D:

> Returned to Caen, I meditated on this result and deduced the consequences. The example of quadratic forms showed me that there were Fuchsian groups other than those corresponding to the hypergeometric series; I saw that I could apply to them the theory of theta-Fuchsian series and that consequently there existed Fuchsian functions other than those from the hypergeometric series, the ones I then knew. Naturally I set myself to form all these functions. I made a systematic attack upon them and carried all the outworks, one after another. There was one however that still held out, whose fall would involve that of the whole place. But all my efforts only served at first the better to show me the difficulty, which indeed was something. All this work was perfectly

conscious.

> Thereupon I left for Mont-Valerien, where I was to go through my military service; so I was very differently occupied. One day, going along the street, the solution of the difficulty which had stopped me suddenly appeared to me. I did not try to go deep into it immediately, and only after my service did I again take up the question. I had all the elements and had only to arrange them and put them together. So I wrote out my final memoir at a single stroke and without difficulty. (p. 89-90)

Although all four examples are experiences of insight, the way these experiences are recounted are not exactly uniform. To see the differences between these four experiences, it is helpful to use a, now classic, segmentation of the phenomenon. It has its origin in a speech that Helmholtz gave on the occasion of his 70th birthday in 1891. Even to him, coming up with new ideas had not been easy. It was only through the "slowly growing from small beginnings through months and years of toilsome and tentative work", that fruitful or happy ideas finally came to him. Only by looking backward he was able to see a much more direct way of climbing the mountain. On his way up, however, he was led on many (unnecessary?) detours. So, like Poincaré (1908/2000) did above, Helmholtz described how happy ideas came to him, here summarized by Wallas (1926/1949) who recognized four stages in this description:

> "He [Helmholtz] said after previous investigation of the problem 'in all directions … happy ideas come unexpected without effort, like inspiration'. So far as I am concerned, they have never come to me when my mind was fatigued, or when I was at my working table. ... They came particularly readily during the slow ascent of wooden hills on a sunny day." Helmholtz here gives us three stages of a new thought. The first in time I shall call Preparation. The stage wherein the problem was "investigated … in all directions"; second is the stage during which he was not consciously thinking about the problem, which I shall call incubation; The third, consisting of the appearance of the "happy idea" together with the psychological events which immediately preceded and accompanied that appearance, I shall call Illumination. And I shall add a fourth stage, of Verification which Helmholtz here does not mention." (Wallas, 1926/1949, p. 52).

Thus, four stages can be distinguished in the process:[2] Preparation, Incubation, Illumination and Verification. These stages can also be recognized in the experiences of Poincaré (1908/2000), the indices in superscript refer to Poincaré's four quotes above.

1) *Preparation,* initially there is a lot of conscious work on the problem without success, expressed as: "every day I seated myself at the work table"[A], "tried a great number of combinations and reached no

2 The exact origin of this distinction in four steps is not entirely clear. Some refer to three stages, saturation, incubation and illumination that Helmholtz would have introduced (e.g., Gell-Man, 1994)⍰⍰. Helmholtz (1908) did not mention the steps in his speech in which Wallas (1926/1949) recognised the steps and named them (which he probably would not have done if Helmholtz had already named them in his text). Wallas' translation of Helmholtz slightly differs (such as: unexpected ~ suddenly; fatigued ~ tired brain; working table ~ desk, used by Wallas and the 1898/1908 translation respectively) albeit this does not seem to affect the meaning. Johnson-Laird mentioned in his foreword to the 1996 edition of Hadamard's *The psychology of invention in the mathematical field (1996, p. x)*⍰ that the origin of the four steps is also often wrongly attributed to Hadamard. However it was Wallas who came up with them. Irrespective of who thought it up, the stages are rather useful here.

results."[A], "apparently without much success"[C], "Disgusted with my failure"[C], "But all my efforts only served at first the better to show me the difficulty, which indeed was something. All this work was perfectly conscious."[D]

2) *Incubation*, the attention is distracted from the problem: "The changes of travel made me forget my mathematical work"[B], "I went to spend a few days at the seaside, and thought of something else."[C], "Thereupon I left for […] so I was very differently occupied."[D]

3) *Illumination*, the solution appears: "At the moment when I put my foot on the step the idea came to me, without anything in my former thoughts seeming to have paved the way for it"[B], "but I felt a perfect certainty."[B], "One morning, walking on the bluff, the idea came to me, with just the same characteristics of brevity, suddenness and immediate certainty"[C], "One day, going along the street, the solution of the difficulty which had stopped me suddenly appeared to me"[D].

4) *Verification,* one has to check it to prevent overhasty euphoria: "I had only to write out the results, which took but a few hours."[A], "On my return to Caen, for conscience' sake I verified the result at my leisure."[B], "I did not try to go deep into it immediately, and only after my service did I again take up the question. I had all the elements and had only to arrange them and put them together. So I wrote out my final memoir at a single stroke and without difficulty."[D]

These descriptions are not fully identical, not all four different experiences of Poincaré (1908/2000) contain all four stages. *Preparation* is explicitly mentioned in experiences A, C, and D; *Incubation* in B, C, and D; *Illumination* in B, C, and D, and *Verification* in A, B, and D. However, in all four stories unexpected insight emerged. Experience D is the only one wherein all four stages are mentioned. *Preparation* is missing in B, however it has been undoubtedly part of the process, in that the problem had to be familiar, well studied, but not yet solved. The same goes for *Verification* that is absent in C. Poincaré's result would not be such a famous finding if it had not been verified. Note that, in experience A *Incubation* and *Illumination* are missing, the focus in this paper is on incubation and moreover on the importance of distraction to influence incubation.

Description A mentions: "I drank black coffee and could not sleep. Ideas rose in crowds; I felt them collide until pairs interlocked, so to speak, making a stable combination". Although the attention might have been somewhat diverted, these phrases suggest that attention was still somewhat occupied with the "ideas" that "rose in clouds" and that there was attention that "felt them collide". In other words the second step of the phenomenon, incubation, is not described. Yet, in the other two descriptions, incubation is rather important for the phenomenon, as shown by the quotes supporting the incubation step. If the incubation step is left out, the phenomenon seems to lose it appeal. This boils down to: There is a lot of work on a problem, without success yet. While working on the problem suddenly the solution appears. The latter is less special and surprising than the experience wherein incubation does play a role. After all that is where thinking is for, to think about or contemplate on a problem and get the solution. Apart from the fact that thinking itself is possible, the special part, in this version of the phenomenon description, is the "get" part. The Illumination step still holds, meaning that the solution appears suddenly: One just gets it... but how? So even in this second description, where attention is not totally away from the problem, the solution still appears suddenly, without a clear conscious path guided by attention leading up to it. Closer inspection, however, reveals that the essence of the incubation step is present in experience A. In other words not a conscious path guided by attention led to the solution, yet there is one! Thus to distinguish four steps still seems appropriate and will be used as a guideline throughout the paper. Poincaré (1908/2000) limited himself to the experiences that were representative for his other experiences of insight. Since there would be no end of collecting all descriptions of experiences of insight, yet it is the existence of all these reports that legitimate a further inquiry of the phenomenon, I have put a couple descriptions from various moments in time in the Appendix. The steps of Wallas (1926/1949) are made explicit for each of them. Like in Poincaré's descriptions steps are sometimes left out. What these descriptions (seem to) share, however, is a common process. Closing here with an important insight of an other mathematician. Andrew Wiles (see, Singh, 1996) describes his experience of a lifelong journey to an answer to Fermat's[3] problem as:

> [...] entering a dark mansion. One goes into the first room, and it's dark, completely dark. One stumbles around bumping into the furniture, and gradually, you learn where each piece of furniture is, and finally, after six months or so, you find the light switch. You turn it on, and suddenly, it's all illuminated. You can see exactly where you were. At the beginning of September, I was sitting here at this desk, when suddenly, totally unexpectedly, I had this incredible revelation. It was the most the most important moment of my working life. (Sing, 1996).

The following chapter includes nine experiments that do use distraction, and five experiments that do not use a distraction period to catch the process leading to insight. The first group of experiments is numbered 1 to 9, and the second group 10 to 14 throughout the whole paper. Each experiment introduced a problem, had manipulation, an outcome and an interpretation of that outcome.

2 Experiments to catch “whatever it is that leads to” insight

2.1 Experiments with a focus on a distraction from the task

1) Patrick's (1938) experiment on “creative thought in scientific thinking” was, according to Fulgosie and Guilford (1968), one of the first experiments on the phenomenon. The participants were asked to come up with an experiment to investigate the relative importance of heredity and environment. There were two conditions: Participants in the continuous-work condition had to plan the experiment

3 Wiles remembered that he, as a 10 year old, encountered the problem in a library book. The problem has been on his mind ever since, albeit he was asked to abandon the project as a student, because the problem had no use (Sing, 1996). The problem of Fermat is the following: $X^2 + Y^2 = Z^2$ is a familiar and widely used formula. There are lots of whole number solutions to this square version of the formula: $3^2 + 4^2 = 5^2$, $5^2 + 12^2 = 13^2$ etc. The problem was to proof whether it is possible or not to have a whole number solution for any other n of the $X^n + Y^n = Z^n$ equations. There is not, and Fermat stated that he had proved it. His proof, however, did not fit within the margins of the book so goes the story and was thus lost or never written down. This footnote has the same problem, for the proof see Faltings (1995).

in one session. During the entire session, that took on average 13 minutes, the participants thought aloud, those thoughts were recorded in a short-hand report. The other group of participants was in the distracted condition. They started with thinking aloud on the problem for 5 minutes. They received a notebook wherein they could jot down, anything they happen to think of. Thereafter they went home and returned after about two or three weeks.[4] They reported their "experiment plan" in a short interview that took about 5 minutes. From this final interview and the first 5 minutes thinking aloud on the problem the experimenter obtained a shorthand report.

In the distracted condition participants were not thinking continuously about the problem, giving opportunity for incubation in at least this condition. Yet, Patrick's (1938) main finding was that the preparation, incubation and Illumination stages were present in both, conditions. Thus, the process in these two conditions was not necessarily different according to this findings. Patrick states that in the continuous-work condition there was

preparation:

> Over two-thirds of the thought changes occur in the first two quarters, which is evidence of the period of preparation, when the ideas are shifting. (p. 60)

incubation:

> [...] was shown if an idea occurred early in the report, recurred one or more times, the subject meanwhile talking of other things, and at last appeared as the chief topic of the experiment. (p. 60)

illumination:

> In each protocol it was noted wherein quarter or quarters the final plan [90% of the time in the 4th quarter] was actually formulated, indicating the third stage of illumination. (p. 61, between brackets added).

For the participants in the distracted condition, the similarity, with respect to the generation of ideas in the first and fourth quarter of the initial 5 minutes thinking on the problem, was seen as an indication that participants were still in the preparation stage. The notebooks, the participants kept, also indicated incubation as "Thoughts about the method recurred from time to time while the participants were occupied with other things". The fact that 75% of the participants gave the final experiment-plan in the final report, and not earlier in the notebook, was evidence for illumination. So, apparently, for Patrick (1938) the time away from the problem suitable for incubation ranges (continuously?) from times-scales of 13 minutes to times-scales of three weeks. Furthermore, as incubation is also present in the continuous-work condition, it is not necessary to be distracted from the problem or to change the subject, to allow a process of incubation.[5]

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4 This difference was not thought important because: "It was considered that a difference of a week or so in an experiment of this nature did not matter greatly, which was borne out by the fact that in several cases, where circumstances prevented collecting the book for a week or two longer than usual, the ratings of the final products were no higher than those of the rest of the group" (1938, p. 59).

5 These observations do not rely on comparison between the conditions. There were also differences between the conditions, for example participants in the distracted condition, on average, planned to use more participants in their experiment plan (132 versus 38 in the continuous-work condition). However this difference did not let Patrick to conclude that one condition did better than the other. Although she expected that since the problem is difficult to solve, with some extra time (two or three months) the distracted condition would do better than the continuous-work condition. This trend was already visible in the data, in the distracted condition 75% of the participants changed their first idea whereas only 50% of the participants in the continuous-work condition did so.

2) Gall and Mendelsohn (1967) used the Remote Association Task (RAT) to investigate creative problem solving. Participants had to come up with a word that connects to three other words, given in sets of three like: {rat, blue, cottage}, {railroad, girl, class}, {surprise, line, birthday}. The solutions of these sets of words are: cheese, working, and party, respectively (Mednick, 1962). Participants were given 30 of these RAT problems. They could work on each problem until they solved it or two minutes had passed. The first five problems that the participant failed to solve were put aside. Thereafter participants in the incubation condition made psychophysical judgments for 25 minutes. It was important that this test was non-verbal, such that it is unlikely to interfere with the verbal process of RAT solving or give "unintended cues", but also demands sufficient attention of the participant to, most likely, prevent thinking on the RAT problems. The participants in the continuous-work condition could think about the RAT problems they missed for a maximum of five minutes or until they solved the problem. Participants in the distraction condition solved, on average, 1.6 of the five problems, whereas participants in the continuous-work condition did 2.9 of the five RAT problems. In the replication experiment these averages where 1.8 and 2.7, respectively. The analyses across the two experiments (included as a factor in an ANOVA) on these differences were significant.

3) Fulgosie and Guilford (1968) asked participants to come up with consequences of two unusual situations: "What would be the result if everyone suddenly lost the ability to read and write?" and "What would be the result if none of us needed food any more in order to live?" (1968, p. 242). The participants in the continuous work condition worked for four minutes on the task. Participants in the distraction conditions worked two minutes on the task, were then distracted for 10 or 20 minutes with a number-series completion task, like: {10, 100, 9, 81, 8, 64, 7, 49, _, _ }. Thereafter the participants worked again 2 minutes on the task. In the continuous-work condition participants noted on average 7.0 consequences, whereas participants who did the 10-minutes distraction task came up with 7.9, and participants in the 20-minutes distraction task condition 9.6 consequences. The distraction condition differed significantly from the continuous-work condition.

4) Dreistadt (1969) used two insight problems, the "farm" and the "tree planting" problem. The "farm" problem pertained to dividing an L shaped farmland in to four parts with the same size and shape. The second problem was to plant 10 trees in 5 rows with 4 trees in each row. The solutions to both problems are presented in Figure 1. Participants worked continuously or were interrupted for 8 minutes, and either received cues or not. The cues were analogies to the problem. The cues were L-formed shapes in the farm problem and star-formed shapes in the tree-problem. Each of the four conditions contained 10 participants, and their solutions were rated on a 5-point scale. So the maximal score was 50. The total scores on the farm problem were: No-incubation-no-analogies (28), incubation-no-analogies (24), no-incubation-analogies (33), and the incubation-analogies (45). The effect of analogy was significant, as well as the interaction between analogy and incubation period, however incubation period by itself was not significant. The scores on the three planting problem were: No-incubation-no-analogies (26), incubation-no-analogies (28), no-incubation-analogies (42), and in the incubation-analogies (47). Although here, the analogy was so obvious (and significant) that the interaction with an interruption period on its own could hardly add to the performance. After the cue it was too easy and participants could not exceed the maximum score anymore. The conclusion was that analogues cues are important for occurrences of insight.

5) Murray and Denny (1969) asked participants to solve the Saugstad “ball problem” (Saugstad & Raaheim, 1957). The task was to move several steel balls from a drinking glass on a movable frame to a metal cylinder. The problem was that both were not in reach of the participant, whose movements were restricted to an area marked with a chalk line. To complete the task the participant could use: A nail, a pair of pliers, a length of string, a pulley, some elastic bands, and several newspapers. The solutions consist of two steps. First the pliers can be used to bend the nail into a hook. The string can be attached to the hook, and now the hook can be thrown to the frame with the glass containing the balls on top, such that it can be dragged toward the participant. Now the participant has to bring the balls to the cylinder. With the papers and the elastic band a telescopic hollow tube can be constructed. The balls could roll through the tube into the cylinder. This task demands the “ability to redefine the stereotyped meanings of objects and events”.

In the continuous-work group, participants worked for 20 minutes on the problem. In the distraction group participants first worked 5 minutes on the problem, then did an unrelated paper and pencil task for 5 minutes, and after that returned to the problem for another 15 minutes or until the solution was found. Murray and Denny suggested that the conflicting results coming form experiments using a distraction period away from the problem might be explained by “the difficulty or complexity of the problem, or, reciprocally, on the participants problem-solving ability”. The latter is used to test this hypothesis. Based on the scores on the Gestalt Transformation Test (a measure of redefinition ability) the participants were divided in a high and a low ability groups, thus forming four conditions. In the continuous-work condition three low-ability participants and nine high-ability participants solved a part of the problem. In the distracted condition this pattern is reversed, eight low-ability and four high-ability participants solved the problem. Neither main effect of ability nor of condition were significant, but the interaction was. They interpreted this pattern of results as caused by a disruption of the status quo of the problem-solving process. For low-ability participants this breaking up of their stereotyped associations was helpful. For high-ability participants on the contrary, this breakup was interfering with their fluent associative process, leading to poorer-problem solving

6) Olton (1979) suggested, that the lack of repeatable results in incubation experiments is due to their designs that “are not reasonable analogues of the situation they were designed to study” (p. 16). To create a design that does match a realistic situation, experienced chess players were asked to solve a couple of “challenging endgame” problems. Participants in the incubation condition were “[...] instructed to 'take a break' at some point of their own choosing around the end of their first hour of work on the task” (p. 16), that fited a natural two hour lunch break. The task indeed “proved to be extremely interesting and highly involving”, and participants were “eager to begin to work on it” (p. 17). Furthermore, post-experimental interviews revealed that, all participants “seemed familiar to the incubation experience, some of them reported having had such an experience with regular chess, and most felt that we had created a situation where incubation might well have occurred” (p. 17). The continuous-work condition kept working on the problem as long as they wanted. Although this appears to be a natural situation to observe the phenomenon, the percentage of participants that solved the problem was the same, 50% in both conditions.

7) Browne and Cruse (1988) used Dreistadt's farm problem. After finding a positive effect of the

distraction task in their first experiment,6 they constructed an experiment with five conditions. Two continuous work conditions spending 20 or 25 minutes on the problem. Participants in the other three conditions first worked 10 minutes on the problem and then spend 5 minutes on one of the three distraction tasks: Analogical hints (graph drawing), relaxation (listening to music with relaxation instructions), or mental-work (memorizing an oceanographic text with the intent of recall). After this break participants worked for another 10 minutes on the problem. So in total these conditions took as much time as the 25 minute continuous-work condition. 17 out of 47 participants in the relaxation condition solved the problem, 10 out of 40 in the hints condition, and 10 out of 42 in the 25 minutes continuous-work condition solved the problem. In contrast, in the metal-work condition 6 out of 39 and in the 20-minutes-continuous-work only 4 out of 42 solved the problem. These latter two conditions differed significantly from the first three. The results were interpreted as showing that the incubation effect was “due to covered problem solving during supposedly off-task time” (1988, p. 183). The higher solution rate in relaxation and in the analogical-cue conditions as well as in the 25-minute-continuous-work condition supported the idea that the extra 5 minutes were beneficial for finding the solutions. In the two worst-preforming conditions, participants had either 5 minutes less time, or were distracted with a memorisation task during those 5 minutes of extra time. Furthermore, after analysing the answers, the experimenters concluded that participants who had used conscious strategies (e.g., recognising relevant cues, -attempting to find a logical problem solving rule, -thinking about the problem during the break). In contrast, none of the participants reported an experience of insight or illumination; thus it might not be an essential part of the phenomenon, but, after all just be covered conscious-problem solving.

8) Dijksterhuis (2004) presented participants four choice alternatives, that is, four apartments for rent. These apartments were described by twelve attributes that could be positive or negative, adding up to 48 pieces of information. The number of positive and negative attributes differed for the four apartments, such that one was the worst (four positive and eight negative attributes), two were medium attractive (Six positive and six negative attributes), and one was the best (eight positive and four negative attributes). The 48 attributes were presented in a random order. Each attribute was presented on the centre of the screen for 4 seconds, automatically followed by the next attribute. The participants were asked to choose the best apartment. Some had to make the decision immediately following the presentation of all four descriptions of the apartments, whereas, others first had to think on it for three minutes. The third group was distracted three minutes prior to choosing the best apartment. They did a 2-back task wherein, they had to decide whether a digit that was presented was the same as a digit that was presented two trials back. This distraction-task was meant to prevent them from thinking on the problem. Each apartment had to be rated on a 10-point scale. The depended measure was the difference between the rating on the best and on the worst apartment. The difference was small for the immediate and the continuous conditions (M = 0.47, SD = 1.71 and M = 0.44, SD = 1.48) and somewhat bigger in the distracted condition (M = 1.23, SD = 2.05) wherein this difference deviated from 0! Dijksterhuis concluded that participants in the distraction condition performed better than participants in the immediate decision condition and those in the conscious thought condition, albeit this effect was “not significant” (2004, see p. 590).

6 In the distraction condition 16 out of 40 participants solved the problem versus 5 out of 40 participants in the continuous work condition.

9) Christensen (2005) asked participants to take the role of an editor of a puzzle magazine, who had to solve eight insight problems within 45 minutes. They had to work fast, but could work on and switch between, the problems any way they wanted. Thus if they got stuck on a problem, they could simply continue with an other problem, and later return to the unsolved one. Every 5 minutes a so called "secretary" put a puzzle on the desk that had priority. The difficulty of the puzzle had to be rated immediately and handed back. These priority-puzzles had their answers included. Some of these puzzles were unrelated to the eight puzzles of the main task, whereas others were analogues to the problems. The solutions to the analogous problems could serve as a cue for one of the eight problems in the main task. A video camera above the table showed which problem the participant was working and at what pace he/she went through the problems. After rating the priority puzzle, the participants returned more often to the relevant (i.e., analogous) main puzzle than to one of the other puzzles. For each participant a resolution score was calculated for analogues cues and unrelated cues. The average of these scores were 0.53 and 0.17 respectively. Thus, after seeing an analogous cue participants were more likely to solve the analogous problem in the main task. This could not be attributed to participants spending more time on the problem with analogous cue, as participants spend 40% less time on analogous cues than on unrelated ones.

In de following section the experiments are compared with the steps Wallas (1926/1949) used to describe the phenomenon. The *preparation stage* is merely the presentation of the problem (Dijksterhuis, 2004; Patrick, 1938), or starting to work on it for a short time (2-10 minutes; Brown & Cruse, 1988; Christensen, 2005; Dreistad, 1969; Fulgosie & Guilfort, 1968; Gall & Mendelsohn, 1967; Murray & Denny, 1969; Olton, 1979). The question is whether the participant really understood or experienced the problem as such. Spending minutes on contemplation on a problem differs most likely from the many days of "toilsome" work that Helmholtz (1890) or Poincaré (1908/2000) spent on it. Moreover Helmholtz and Poincaré made very clear that they got stuck on their problems. This particular standstill and its duration was suggested by Wallas as a necessary condition, but apparently was given little consideration in the above reviewed experiments.

In contrast to the approach to the preparation step, the experiments did all concentrate on the *incubation step*. The incubation step was operationalized by differentiating between the conditions of the experiment in such a way that in one condition participants were distracted and in the other condition they were not. However, despite this straightforward manipulation, the results across the experiments were not consistent. Both Patrick (1938) and Olton (1979) did not find a difference between the continuous-work and the distracted conditions. In other experiments performance in the distracted condition was better (Fulgosie & Guilford, 1968; Dijksterhuis, 2004), but only in combination with analogue cues (Christensen, 2005; Dreistadt, 1969) or only for low-ability participants (Murray & Denny, 1969), albeit distraction was beneficial it was only due to the extra time (Browne & Cruse, 1988). In other experiments participants in the continuous-work condition did better (Gall & Mendelsohn, 1967). All these different results arise from basically the same method: Spend time away from the topic as a way to catch insight. However, it appears to be not that simple. Table 1 based on data from Sio and Ormerod (2009) reveals that for various times ranging from 3 minutes to 1 day incubation effects are reported, although other experiments using incubation times that fell within the same range failed to find the same effect. Apparently, there does not seem to be a right time, required for incubation to occur.

Another issue is the absence of the *illumination step*. In none of the experiments there is a notion of illumination described: The problem is solved or unsolved but it remains unclear how the participant solved the problem or whether there was an accompanying experience of suddenness and clearness involved.

To end, the *step of verification* is not given special notice. Verification is differently interpreted by the experimenter as by the participant. The experimenter focuses on receiving an answer from the participant and when this goal is achieved, the experiment is finished. By contrast, the participant (should) want to know whether his answer was correct. Irrelevant, whether it was a guess or an illumination, the goal is only reached if the problem is indeed solved. However, the opportunity to verify is rarely given.

All stages are clearly present in the descriptions of Poincaré (1908/2000), Kekulé (1890) or Wiles (1996), who worked on their problems for years. During this period they must have gathered lots of ins and outs concerning their problem. Their experiences are also characterised by a moment of illumination, which made it immediate clear to them that their ideas were new and important! However, the experiments above just concentrated on distraction, and it remains unclear whether the other steps came along. What did become rather obvious was that these other steps were not considered to be important all along. The experiments in the following section search for insight but do not use a period of distraction as part of their method.

Table 1. Various incubation times, that had an effect (left column) and did not have the expected effect (right column).

Incubation times that had Effect		Incubation times that had no Effect	
Author	Time (minutes)	Author	Time (minutes)
Mednick et al. (1964, exp. 1 & 2)	?	Dominowski & Jenrick (1972, exp. 2)	3
Yaniv & mayer (1987)	?	Dorfman (1990, exp. 4)	3
Christensen & Schunn (2005)	?	Brown & Cruse (1988)	5
Smith & Blankenship (1991, exp. 4)	0.5	Jamieson (1999, exp. 1 & 2)	5
Peterson (1974)	1.8	Vul & Pashler (2007, exp. 1)	5
Sio & Rudowicz (2007)	2	Dorfman (1990 exp. 4)	8
Segal (2004)	4	Torrance-Perks (1997, exp. 2)	8
Murray & Denny (1969)	5	Dominowski & Jenrick (1972, exp. 1)	10
Patrick (1985)	5	Bennett (1975)	10
Smith & Blankenship (1989, exp. 2)	5	Torrance-Perks (1997, exp. 1)	10
Smith & Blankenship (1989, exp. 4)	5	Hansberry (1998, exp. 3)	10
Dorfman (1990, exp. 3)	5	Fulgosi & Guilford (1968)	10
Smith & Blankenship (1991, exp. 1 & 2)	5	Dorfman (1990 exp. 4)	13
Smith & Blankenship (1991, exp. 4)	5	Olton & Johnson (1976)	15
Vul and Pashler (2007, exp. 2)	5	Hansberry (1998, exp. 2)	15
Seabrook & Dienes (2003)	7	Beck (1979)	20
Dreistadt (1969)	8	Goldman et al. (1992)	20
Smith & Blankenship (1989, exp. 3)	10	Gall & Mendelson (1967)	25
Houtz & Frankel (1992)	10	Kaplan (1990, exp. 3)	30
Medd & Houtz (2002)	10	Fulgosi & Guilfort (1972)	60
Segal (2004)	12	Penney et al. (2004, exp. 1)	180
Smith & Blankenship (1989, exp. 2)	15	Silveira (1972, exp. 3)	210
Smith & Blankenship (1989, exp. 3)	15	Henley (1999, exp. 3.2)	1140
Dorfman (1990, exp. 3)	15	Henley (1999, exp. 4)	1140
Dodds (2002, exp. 1 & 2)	15	Penney et al. (2004, exp. 2)	1440
Moss (2002)	15		
Penney et al. (2004, exp. 1)	15		
Penney et al. (2004, exp. 2)	15		
Brockett (1985)	20		
Fulgosi & Guilford (1968)	20		
Kaplan (1990, exp. 4)	28.8		
Silveira (1972, exp. 1)	30		
Kaplan (1990, exp. 1)	30		
Penney, et al. (2004, exp. 3)	30		
Fulgosi & Guilfort (1972)	30		
Beck (1979)	30		
Kaplan (1990, exp. 2)	32		
Penney et al. (2004, exp. 3)	120		
Silveira (1972, exp. 1)	210		
Goldman et al. (1992)	1440		

2.2 Insight as a change of focus

10) Maier (1930) asked participants to construct two pendulums that could put two chalk marks on two spots indicated on the ground. To do this they could make use of the following materials: 2 poles of 1.9 meters, 2 other poles each about 1 meter long, 1 table clamp, 2 burette clamps, 2 pieces of electric bell wire about 2.3 meters long, 8 pieces of lead tubing 1cm in diameter and from 5 to 15 cm long, and several pieces of chalk. The difficulty was that there was not enough material to tackle the problem considering the most obvious solutions.7 The experiment had *five conditions*: participants in the *control* condition only received the problem instruction before they started to work on the problem. Participants in the other conditions received, in addition to the problem instruction, three construction experiences (i.e. cues) and or a "direction" (i.e. angle of approach or perspective) that could guide to the whole solution. The construction experiences showed how some of the parts could be used together. Such as A) "making a plumbline" (using Burette clamp as weight and wire), B) "combining poles" (using the table clamp and two poles to lengthen the poles), C) "wedge" (using two poles forming a stable T-structure). In the *second* condition these experiences were given before and in the *third* condition after the problem instruction. Participants in the *fourth* condition did not get the three experiences but got a guiding experience: "direction" instead. They were told "[...] appreciate how simple this problem would be if we could just hang the pendulums from a nail in the ceiling. Of course, that it is not a possible solution but I just want you to appreciate how simple the problem would be if that were possible. Now that it is not possible the problem is, as you may find, really quite difficult" (Maier, 1930, p. 119). Participants in the *fifth* condition were presented with all three experiences as well as direction.

In two different experiments of Maier (1930), in Berlin and in Michigan, participants in condition five (with the three "experiences" and the "direction") preformed the best; four out of ten and four out of twelve correct solutions respectively. Thus taken together eight out of twenty-two. Only one participant, from the other four conditions, found the correct solution, out of 62 participants in these conditions.8 After several failing attempts sometimes direction was given and seemed to have a beneficial effect on finding the solution.9 This hint let participants to the insight that the ceiling could be used as a part of the construction, without which the individual experiences would not have been useful. The table clamp could be used to combine a long and a short pole (A) to be long enough to wedge the other long pole flat against the ceiling, forming a T-structure (C). From both ends of that pole the electric belwire could be hung down, with a piece of chalk attached to it with the burrete

7 Four poles are not enough to make two three-pod's as stable self standing structures. The long poles can not reach the ceiling. The short pole can be attached to the long pole to reach the ceiling with the table clamp, however there is only one, and two pendulum attachment points are needed. Attempts to wedge the poles between the walls also did not suffice to create a stable structure.

8 Total number of participants in the two experiments was 84, 52 in Berlin and 32 in Michigan. Not all conditions were included in both the experiments, in Berlin condition four and in Michigan conditions one and two were omitted.

9 The "direction" was given to eight of ten University of Michigan students in condition three after they had failed in solving the problem. (Condition three had been given parts A, B, and C with the instructions to use them.) Almost immediately two of the participants had the solution; two others returned to one of their old ideas, but got the solution when the "direction" was repeated a little later. The remaining four participants seemed to receive no benefit from the "direction".

clamp. Only participants who received the three experiences and the direction that guided how to combine the experiences (condition five), could use them to get the solution. The experiences showing how the material could be used, were important but not sufficient by itself. A direction of how to bring these individual experiences together to a combined construction was needed in addition to change the perspective on the problem, but was by itself insufficient too.

11) In a following experiment Maier (1931) used a slightly different problem. Two cords were hung from the ceiling. The task was to tie the ends of the two cords together, but it was impossible to simply grasp both cords. To solve the problem, participants could use any object in the room, which contained: poles, ring stands, clamps, pliers, extension cords, tables and chairs. These material suggested several solution, such as: using a pole to pull one of the cord, an extension cord to lengthen a cord, the table to hold one cord in the middle. Apart from these obvious solutions, it was also possible to use the pliers to attach a weight to a cord and put it in motion. The last solution was a lot more difficult to find.[10] After finding a solution, the participant was told to think of another one, until the participant had discovered the more difficult solution. If a participant could not find another solution for 10 minutes, the following hint was given, without explicitly informing the participant about it. The experimenter went to the window, and when passing, walked against the cord, putting it in motion.[11] Maier did not create any difference in conditions between participants but observed what exposure to the problem elicited in participants. Accordingly, he afterward divided them into three groups, namely, 24 participants who found the solution without the hint, and 14 participants who did not find a solution at all. The third and most interesting group of 23 participants found the solution soon after the hint was given. The time that elapsed between the effective hint and the solution was very short, on average 42 seconds. However, these participants did not always consciously perceive the hint as contributing to the solution! As a result of this observation Maier split up the third group into two subgroups. The first subgroup saw the solution at once as a whole. They did not recall the hint as being important for the insight. For the second subgroup, the solution came in two steps, that is seeing the cord as a pendulum, and seeing the pliers to attach weight to the cord in order to use it as a pendulum. In the latter subgroup all except one participant reported that the hint had led them to the solution. Furthermore, when the solution was shown to participants who failed to solve the problem, they immediately saw that the hint should have made them see the solution. Hence they could not understand how they failed to see it. That is why Maier thought it was quite unlikely that the hint was unimportant for participants or that they did not wish to admit its importance. Furthermore, he considered the possibility that the hint might not have been experienced because of sudden illumination dominated consciousness. In short, the conscious memory of passing through the solving steps disappeared. Accordingly participants who found the solution without a hint could not tell how they came to it, and explained it as: "It just dawned on me”, "It was the only thing left'', "I just realized the cord would swing if I fastened a weight to it", "Perhaps a course in physics suggested it to me", and "I tried to think of away to get the cord over here and the

10 “The easier solutions thus functioned as (a) a means for encouraging the participant; (b) a period of orientation and adjustment to the situation and the experimenter; and (c) a means of stating the problem without limiting the freedom of the participant.” (1931, p. 181)

11 If this did not lead, in a few minutes, to a solution a second hint was given, the participant was given a pair of pliers and was told: "With the aid of this and no other object there is another way of solving the problem". This second hint will not be discussed any further in this paper.

only way was to make it swing over" (1931, p. 188). In the beginning participants perceived the materials as static objects, but after the hint was given the perception of the material possibly changed rigorously. Making the cord do something was unusual, because suddenly it could be seen as a pendulum rather than a cord hanging from the ceiling. This change in perception of the materials' applicability appeared together with a different way of looking at the problem. The hints made the pendulum idea better accessible, just like “additional points represent the organization of a circle more readily than three points” (1931, p. 191). Maier suggested that the swinging of the cord could have changed the way successful participants perceived the problem and what was regarded as the difficulty.

12) Another approach to insight comes from Luchins (1942). He used three conditions wherein he either offered a hint or not (condition one and two), and a third condition wherein he varied the type of problems. Participants received three jars with different amounts of water and had to end up with a precisely required amount of water. In the first two conditions the five problems following the example (E1 to E5) could be solved in exactly the same way (see and try these and the following problems in Table 2). Most participants did so and became habituated to this particular problem solving approach, Luchins called this einstellung. Participants exclusively had to fill the middle jar, and take out two fills of the right and one of the left jar, which resulted in the required amount of water in the middle jar. This solution would also work for the two problems that followed (C1 and C2), although these could also be solved in a simpler, more direct way. The seventh problem could simply be solved by filling the right jar from the left jar, and the eight problem could be solved by adding water from the right jar and from the left jar to the middle jar. The strategy that worked for the first eight problems was not always the most efficient. And altogether it did not work for the ninth problem, which could only be solved by filling the right jar from the left jar. Problem ten and eleven again could be solved, either in the habituated way (suitable for the first eight problems), or also in a more direct way.

Table 2.
The einstellung (E1-E5) problems, followed by two sets of critical (C1-C4) problems

Problem		**Given the following empty jars as measures**			**Obtain the required amount of water**
1		29	3		20
2	E1	21	127	3	100
3	E2	14	163	25	99
4	E3	18	43	10	5
5	E4	9	42	6	21
6	E5	20	59	4	31
7	C1	23	49	3	20
8	C2	15	39	3	18
9		28	76	3	25
10	C3	18	48	4	22
11	C4	14	36	8	6

The second condition differed from the first in that they received a hint. At the start of the experiment these participants were asked to write the words “don't be blind” next to the sixth problem. Due to this hint the participants used more direct solutions for problems seven and eight in condition two (44%) than participants in condition one (17%). In other words participants in condition two were less habituated to the standard problem-solving approach that was used for problems two to six. Furthermore after problem nine the number of direct solution increased, for problems ten and eleven, in both conditions (44% and 70% for conditions one and two respectively).[12] In short, providing a hint and/or being confronted with problem nine both contributed to a shift towards a more direct solution approach. The control group started directly at problem seven, all these participants (100%!) used the direct approach to the solution. The participants who started with problem two, however, nearly all kept using the approach that was suitable for problems two to five. Just a few participants used the more direct approach for problems seven and eight. After encountering problem nine that could not be solved with the habituated approach and therefore had a disturbing effect, the number of direct solutions increased for problems ten and eleven. In short, problem nine turned out to elucidate the limitation of the habituated problem-solving approach. To verify this provisional conclusion, Luchins repeated these experiments several times in a number of different groups, using different types of material. He found similar results. So, the problem-solving approach that worked fine for the first eight problems, guided participants to a way of looking at the problems that made it harder to see a more direct approach. Of course the participants were not incapable of solving problem nine. The habituation created the difficulty. This was confirmed, when participants, after completing the experiment were shown the more direct solution, they spontaneously said: "How dumb I was"; "How stupid of me"; "How blind I was"; or made other similar comments (1942, p. 2). This experiment of Luchins is a prime example of how important the context (habituation) is for the approach of a problem.

13) Kaplan and Simon (1990) asked participants to decide whether or not an eight by eight grid, of which the right under and the left upper corner were cut out, could be covered with 31 domino stones in such a way that each could cover two blocks of the grid. The solution had to be a decision (yes or no) supported by a “rough proof”. Although the description of the problem (and, in hindsight also the solution) was rather simple, the problem actually was not. The “magnitude of problems difficulty” was estimated by the size of the search space that is the number of possible coverings of the grid. This option is not available for humans since “A computer program required 758,148 domino placements in order to prove the problem impossible by exhaustion” (1990, p. 379).[13] An important step towards an alternative solution is recognizing that only neighbouring squares of different colours can be covered with a domino stone. This means that there had to be as many black as white squares. Seeing

12 These percentages are the averages across six experiments (table 1. from Luchins, 1942, p. 6) The range around the percentages and their standard deviations were in condition one (Total *N* = 79) for problems seven and eight: *Min* = 0%, *Max* = 30%, *SD* = 10,41 and for problems ten and eleven: Min = 30%, Max = 63%, SD = 12,45. In condition two (Total *N* = 86) they were, *Min* = 16, *Max* = 36, *SD* = 19,86 and *Min* = 54, *Max* = 87, *SD* = 12,81 respectively. There were 57 participants in the control group.

13 How difficult the problem can be is illustrated by the following quote “a graduate student in Chemical Engineering spent 18 hours and filled 61 pages of a lab notebook with notes, yet still did not solve the problem! While the notebook contains numerous drawings of boards and potential domino placements, the boards were never drawn with alternating squares shaded differently. As the student was given only a written description of the problem, and not an actual checkerboard, presumably the color of the squares was not available to him unless he shaded them himself” (1990, p. 379-380).

this parity rule was thought to depend on the way the participant saw the problem. In particular the problem space wherein the participant looked for a solution as well as the form of a solution he expected, appeared to be of critical importance. The problem was presented as an eight by eight grid in four ways that differed in the salience of parity: a blank grid (1), a grid coloured black and pink in a checker board fashion (2), the words "black" and "pink" alternating written in the squares of the grid (3), or the words "bread" and "butter" alternating written in the squares of the grid (4).

To facilitate the solution process, hints were given at various times. If after 15 minutes, a participant was still trying to cover the board he was told that this was impossible and asked to provide a rough proof for this impossibility. If the problem was not solved after another 15 minutes, participants were told that the colours and words go in pairs (the parity hint). If this hint did not lead to a solution within 10 minutes they were asked to count both colours. None of the seven participants in the blank-grid condition could solve it without hints. The hints apparently functioned as positive constraints assisting participants to constructively limiting their search space. Participants in the black/pink-colour (one out of five) and in the black/pink-written (two out of six) conditions did perform slightly better. Finally three out of five participants in the bread/butter condition solved the problem without hints.

When comparing the conditions, a remarkable finding concerned the discrepancy between the time participants needed to detect parity and the duration until they began to implement parity in their solution approach. Participants in the blank and in the black/pink-coloured conditions took longer to recognize parity than participants in the blank/pink-written and the bread/butter conditions. Nevertheless, in de latter two conditions it took participants longer to finish the problem *after* they reported recognition of parity (on average 473 and 653 seconds respectively) than it took participants in the blank and the black/pink-coloured conditions (who needed, on average, 262 and 110 seconds respectively). Moreover the time differences between conditions became negligible after looking at the time participants needed to give a rough proof when a) they had discovered the impossibility of the covering-solution and b) they then mentioned parity as an alternative strategy. A conclusion Kaplan and Simon (1990) draw on these results was that participants only looked for another approach if they were dissatisfied with the current approach and dispose of appropriate cues to get directed to a new approach. Although participants in the black/pink-written and the bread/butter conditions mentioned parity (a potential cue for a new approach) pretty early, the exhaustive covering approach was still promising at that moment. So, they did not yet need to look for a new approach.

14) Stephen, Dixon, and Isenhower (2009) presented gear systems of four to eight connected gears to the participants. Given the turning direction of the driving gear, participants had to decide which way the target gear would move: clockwise, counter clockwise, or that the system would "jam" and thus not move. Participants had to solve 36 of these problems, each one presented in a trial. Of all participants 84% initially traced the direction of the gears with their finger, on which a motion tracking marker was fixed. During the experiment most participants (61%) discovered a simpler way to solve the gear problems: when the number of gears is even, the target gear turns in the opposite direction of the driving gear, if the number of gears is odd the target gear moves in the same direction as the driving gear. After finding this "alternation" or "parity" rule the use of the finger changed, in that it was no longer necessary to follow the gears driving direction, but counting the gears would do.

The accuracy did increase after this discovery (from 77% to 90 % correct). After that, the function of the finger movements changed. So only the finger movements of the last five trials before discovery were used for analysis. Initially, the variability in finger movements, measured as entropy was rather stable, which could be interpreted as that the participants had a stable representation of the problem and of its solution. In other words, participants tried to follow the gears but did this with a certain inaccuracy. Finger movements and the representation of the problem, seen as two different systems, became loosely coupled. But since this coupling was not perfect the variability in finger movements contributed to an increase in instability of the representation. At some point, this instability became magnified to such a degree that the representation broke down (decrease in entropy). As a result a new cognitive structure, that is, insight emerged, at least within those participants who solved the problem. To summarise, a peak of entropy followed by a decrease of entropy are indicators of a phase transition that comes along with insights.

In the following section I will integrate the five aforementioned experiments into the four steps described by Wallas. In all these experiments, the *preparation step* was more or less congruent with working on the problem. That means, participants tried, failed and tried again, and at some point saw the light. A couple of aspects were characteristic for this stage. For example trying and finding solutions was a way to get familiar with the problem and to progress to the (more difficult target) solution (Maier, 1931). Stephen et al. (2009) used this step to give participants a chance for adapting to another view on solving the problem. Furthermore, the kind of information that participants received before starting to work on the problem varied. Hence the availability of this information changed the way the problem was perceived in terms of difficulty (Kaplan & Simon, 1990; Maier, 1930). In Luchin's (1942) experiment the habituation to a particular problem solving-strategy created the trouble. Finally, almost all experiments (except for Stephen et al., 2009) provided cues or hints to reduce or constrain the diversity of looking at the problem. To sum up, the preparation step was less explicitly marked as being a separate step as in the distraction experiments. Participants just continued working[14] on the problem and probably understood and experienced the problem as such, which is in sharp contrast to the limited preparation step in the distraction experiments. However, again it is unclear how well these problem presentations resemble the difficulties Poincaré, Kekulé and Wiles experienced.

The *incubation step* is not as explicit as in the experiments that used a distraction task, but it probably was not absent either. Although the participants were continuously working on the problem, they did not experience the solution as a result of this continuous work. When asked they could not contemplate on how they arrived at the solution (Maier, 1931). Just like after typical incubation (spending time away from the problem), the solution came unexpectedly! In hindsight, it seems that while working on the task a new way of looking at it emerged. Interestingly, participants were only aware of the outcome but not of the underlying process that led to it. From this we may conclude that although, in many of the experiences there is time spent away from the problem (i.e., Poincaré's experiences B, C and D), this might not be a necessary condition. The reason is that insight also occurred in Poincaré's experience A and in the experience that Kekulé described, as well as in the experiments 10 to 14, despite the fact that attention was not completely away from the topic.

14 Based on the distraction experiments, insight should not be expected to occur under these conditions.

The insights that suddenly and unexpectedly emerged correspond with Wallas' step of *illumination*. Stephen et al. (2009) concentrated on the sudden shift from an initial perspective to a new perspective on the problem. Kaplan and Simon (1990) noted that the experience of this shift was characterised by a rapid sequence of thoughts and by the feeling of being on the right track. Both thoughts and feelings proceeded so quickly that there was hardly time to make them explicit. Maier (1930, 1931) concluded that it was rather impossible to give a description of the reasoning process that led to the solution. Furthermore, the explanations given by participants were not always accurate (Maier, 1930). When shown the solution, participants instantly understood it and could not understand that they had not seen it earlier (Luchins, 1942). In hindsight, it was difficult to conceive how the initial perspective on the problem was taken in to account in the first place.

In contrast with most experiments using distraction, the *verification* step was an integral aspect of the task (solving the problem), although it was not explicitly intended as such. Only when they had solved the problem, participants were able to make the solution explicit (but not the process) by giving a rough proof (Kaplan & Simon, 1990). In the experiments of Maier (1930, 1931), the verification step was accomplished when the solution worked out. Luchins (1942) gave feedback to his participants about the correct answers. To conclude, in all experiments (1 to 14) no special attention was paid to the verification step, apart from the experimenters who verified the correctness of the answers.

3 Conclusion

In the last paragraphs of this paper, I have reviewed a series of experiments that attempted at investigating the phenomenon of insight. The first description of the phenomenon as Poincaré experienced it dated from 1908, about 27 years after the actual event took place. It also took 28 year until a written version of Kekulé's experience reached the public.15 Wiles' experience dates from 1994 and was broadcasted on BBC two years later. The time gap between the moment of insights and the publication of these subjective experiences creates some uncertainty about the accuracy of the portraiture. Retrospectively, it is difficult to determine whether important aspects of the phenomenon may have remained unnoticed while less pivotal aspects may have attracted too much attention. To make the experiences more comparable with each other, the steps of Wallas (1926/1949) were introduced and applied to the experiences as well as the one's described in the Appendix. As in the descriptions of Poincaré, not all steps are always equally clearly present in every experience. However, the descriptions do rather resemble one another and it seems permitted to say that they do describe a fairly common experience. Thus, although the description of the experiences is often characterised by the four steps of Wallas (preparation, incubation, illumination and verification), the process underlying the phenomenon is not per se a reflection of those steps. This process could also be ascribed (to an unknown degree) to the interaction of components that have been undetected so far. In other words, investigating the phenomenon on the level of describing the

15 The story was told many times to friends and family before it was published in 1890, and there is also a version with monkeys holding hands and dancing in a circle published in 1886 (Rocke, 1985).

experience does not necessarily mean that this level is appropriate to reveal which mechanism is responsible for the phenomenon.

The four steps of Wallas (1926/1949) were also applied to compare Experiments 1 to 14 with respect to the mechanism underlying the phenomenon. The experiments differed quite remarkably to the extent that they paid attention to the steps. There were no experiments with all steps present. To recap Experiments 1 to 9. Despite the presence of the preparation step it was so short that it could not nearly equal or be comparable with the preparation of, for example, Poincaré's experience (1908/2000). By contrast, all experiments focussed on distraction, whereas illumination and verification were given no interest. A slightly different picture occurred with respect to Experiments 10 to 14. Because the preparation step was more or less congruent with working on the problem, preparation was present in that sense. Yet as a step itself, it was not explicitly mentioned by the experimenters. Incubation was even less recognizable, but I argue that it was there, because it is typical for incubation that the solution follows unexpectedly. Illumination again was mentioned, but verification did not receive special notice in these experiments. Interestingly, a range of outcomes was found over the years in experiments that tried to establish the existence of the phenomenon (e.g., Experiments 1 to 9). This variety of findings hampered drawing an unequivocal conclusion. Brown and Cruse (1988) ascribed the lack of conclusive arguments to “the plethora of methodological factors” (p. 178), such as, insufficient power, inadequate control groups, wrong variable choice and so on. One question that should be asked at this point is what kind of methodological approach would then be appropriate to investigate the (existence of the) phenomenon.

A rather straightforward and well-conceived view on measurement was proposed by Borsboom et al. (2004). They argued that a construct could only be measured if a) it exists and if b) variation in the constructs would cause variation in the measurement outcome. With regard to the experiments (i.e., 1 to 9) that used incubation, except for Christensen (2005) who linked analogue and unrelated cues to the solution rate, the measurements focused exclusively on the outcome, that is, whether or not the problem was solved at the group level. The main assumption was that insight (illumination) requires incubation, but verification of the occurrence of insight did not take place, on an individual level. The success rate was determined by comparing the group averages of the control and the experiment group. The variation in outcomes was reduced to three possible value states, that is: The phenomenon exists, it does not exist, or the existence is undetermined. To obey the measurement principles of Borsboom et al., a variation in the outcome score should be causally linked to a variation of some part(s) of the phenomenon. Note that it had to exist in de first place. To sum up, what is missing in Experiments 1 to 9, is a clear description of how to measure the phenomenon. To begin with, its relevant attributes have to be determined. A significant quality of these attributes is that they have to be detectable. For example, incubation only gets noticed if it is followed by illumination. Apart from that, it remained indistinctive whether providing the correct answer was accompanied by the experience of illumination. Yet, illumination, that is the sudden and unexpected experience of insight, is the key step of the phenomenon. By lacking to establish the link between the outcome and the occurrence of insight (illumination), it becomes questionable whether these experiments investigated the phenomenon at all.

This apprehension lessens in urgency when reflecting upon Experiment 10 to 14 in the light of the measurement principles formulated by Borsboom et al. (2004). Their approach to the phenomenon was geared to find the crucial aspects that led to its appearance. For that reason, in Experiments 10 to 14, the engagement of participants in the problem-solving process was narrowly observed, hardly limited by time constraints. The experimenters were rather focussing on the moment supreme, that is, the moment when participants reported insights or when this experience became visible. Maier (1930, 1931), Luchins (1942) and Kaplan and Simon (1990) searched for a link between variation in the outcome and the manipulation of a factor, for example, providing cues or varying the (problem) conditions. However, their effort focussed mainly on the outcome and not on the process leading to it. Although the personal experience of participants was acquired by asking them about it, the success rate of getting insight was established by calculating group averages. In short, these experiments made a promising start to catch the mechanism underling insight. Yet, the (individual) process leading from the state of unknowingness to the state of illumination (phase transition) was not captured, because their measurements did not focus on that process. Moreover their measurement was not appropriate to measure the process, even if they wanted to do so.

Processes, like phase transitions that occur in non-linear dynamic systems can be delineated by parameters of non-linear models (e.g., catastrophe models). For example, discontinuous changes as occurring when water turns into steam due to a raising temperature are exhaustively described by catastrophe models, that are a part of non-linear system theory (van der Maas & Molenaar, 1992). A cognitive state, like sudden insight, can be thought of as a result from a discontinuous change in cognition. That is, the suddenness of which insight is noticed, points to a non-linear transition underlying it. From that perspective, the process of gaining sudden insight could be compared to Piaget's stage theory of cognitive development, which is characterized by discontinuous changes. Van der Maas and Molenaar (1992) used the catastrophe model to explain the sudden jumps occurring when children cognitively progress from a more simple state (non-conserver) to an advanced state (conservers). Non-conservers, that is, children who do not understand the principle of conservation (e.g., the amount of water poured from a small tall container into a wide low container does not diminish or increase), do make predominantly mistakes in judgment due to an inappropriate strategy. When children progress to the transitional phase, they strongly oscillate between inappropriate and appropriate strategies, resulting in wrong and right answers, which indicates that no stable pattern is yet established. However, at some point, the oscillation frequency suddenly decreases steeply and a final state (conservers) is reached wherein hardly any mistakes are made.

In line with the application of the catastrophe model on stagewise cognitive development, it could be assumed that dynamic parameters such as applied in physics (e.g., power law, entropy etc.) are suitable to investigate the phase transition leading to illumination. This possibility prompted Stephen et al. (2009) to hypothesize that those parameters were appropriate for investigating the phenomenon. Their experiment made use of real-time measures, by registering the exact moment of reaching insight of each participant. The behaviour of the parameters was causally linked to the participant's experience of insight. Evidence for this claim is found in that the experience of insight always occurs instantly after a peak in entropy followed by a steep decrease. In contrast to all other reviewed experiments, this pattern was applicable to all participants, in other words, variation in the constructs would cause variation in the measurement outcome, as proposed by Borsboom et al. (2004).

So far, in this paragraph, I have debated that there is still little consensus about what the phenomenon exactly is and what the appropriate way would be to investigate it. Stephen et al. (2009) used a measurement (e.g., entropy, power law) that they borrowed from physics. It provided them with the possibility to make predictions about the outcome of their measurement. The absence of such a framework impeded finding unequivocal explanations of the process due to the complexity of factors involved in the process, as brought up by Brown and Cruse (1988). Apart from that, missing a theoretical embeddedness led to an inflation of 'explanations' that were basically nothing else than naming the unknown process (e.g.,'unconscious cerebration', 'incubation', 'unconscious thoughts' etc.) between measurements (see Carpenter, 1876; Patrick, 1938; Gall & Mendelsohn, 1967; Dijksterhuis, 2004). Therefore, the explanations provided by the research discussed above (with the exception of Stephen et al., 2009) do not derive from systematically varying a certain factor to demonstrate that this factor has an effect on the outcome as predicted by a theory.

When theory is absent, the urge increases to look even more closely into the meaning of a certain phenomenon. In line with the experiment of Stephen et al. (2009), one can assume the phenomenon has to be studied over time to understand why some participants get insight at some point and others do not. Poincaré (1908/2000) expressed these thoughts by asking himself: Why is not everyone capable of learning mathematics? He concluded that something more must be going on than just following rules, because although many people are able to do so, they still do not succeed in understanding complex mathematics. Poincaré came up with an interesting analogy, referring to the movements of atoms. In complete rest, atoms, like thoughts would be totally immobile and therefore not in touch with each other[16]. However, in relative rest, for example during incubation, certain atoms or thoughts were moving, eventually bumping into each other. As a result of this clash, new structures of thoughts or insight could emerge. These new structures were probably not the result of a random process because interesting new thoughts implied coherence and logic. The moving atoms differed from motionless ones in that the owner has paid close attention to them. Poincaré concluded that not everyone has, to the same degree, the ability to pay attention to (moving) atoms. I argue, that this ability to pay attention can be thought of as self-perturbation (e.g., perturbation generated by a person's own actions) and, that both self-perturbation and externally caused perturbation are picked up in rather different ways. The perturbation that is needed to progress from a state of relative unknowingness to an advanced state (of knowingness) depends on the individual and his er her embeddedness in a context that varies over time.

To conclude, the phenomenon is far to complex to be investigated by studying isolated factors, since there will always be a difference in (initial) conditions that cannot be controlled for. Choosing a factor and manipulating it to see whether it effects an outcome measure, of which it is unknown how it precisely relates to the phenomenon, is a vain endeavour. Therefore, I propose that before conducting experiments, there should be a theoretical framework (e.g., catastrophe theory, complex system theory) about what the phenomenon is and how it manifests itself. Based on this knowledge, a measurement should be decided upon that takes individual differences in experiences into account when investigating the process that leads to sudden insight.

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16 Meanwhile, it is common knowledge that atoms come to rest only at a temperature of 0º Kelvin.

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Appendix

The phenomenon described

A couple of descriptions of the phenomenon, of insight after not being conscious of thinking on the problem the moments before. The steps of Wallas, (1926/1949) *preparation, incubation, illumination,* and *verification* are made explicit for each of them. It can be verified that the descriptions express similar ideas for each step. Verification is often missing in the descriptions, from a couple of descriptions it is save to say that they would not have been described if the insight did not work. Moreover Poincaré (1908/2000) and Kekulé (see Benfey, 1958) both stress the importance of verification.

J. G. Sulzer (1720-1779)

> At times thoughts will not develop or let themselves be clearly grasped while we pay full attention to them. Yet long afterwards they will appear, of their own accord, in the greatest clarity just when we are not in search of them, so that it seems as though in the interim they had grown unnoticed, like a plant, and now suddenly stand before us in their full development and bloom. Many a conception ripens gradually within us until, freeing itself from the mass of obscure ideas, it suddenly emerges into the light. (1774 in Rand, 2004, p. 284).

Preparation: "At times thoughts will not develop or let themselves be clearly grasped while we pay full attention to them".
Incubation: "Many a conception ripens gradually within us", "just when we are not in search of them".
Illumination: "long afterwards they will appear", "of their own accord, in the greatest clarity", "freeing itself from the mass of obscure ideas", "suddenly emerges into the light".
Verification: -

Carl Friedrich Gauss (1777-1855)

> "Finally, two days ago, I succeeded, not on account of my painful efforts, but by the grace of God. Like a sudden flash of lightning, the riddle happened to be solved. I myself cannot say what was the conducting thread which connected what I previously knew with what made my success possible." (in Hadamard, 1945, p. 15).

Preparation: "painful efforts".
Incubation: "cannot say what was the conducting thread".
Illumination: "by the grace of God", "Like a sudden flash of lightning", "the riddle happened to be solved".
Verification: -

Griesinger (1817-1868)

> There is in intelligence an actual, though to us an unconscious life and movement; we recognise it however by its results, which often suddenly make their appearance from some unexpected source. A constant activity reigns over this almost, if not wholly, darkened sphere, which is much greater and more characteristic for the individuality than the relativity small number of impressions which pass into the state of conscious. (in Whyte, Lancelot, 1960, p. 160).

Preparation: -

Incubation: “unconscious life and movement”.
Illumination: “often suddenly”, “make their appearance”, “from some unexpected source”.
Verification: “recognize by its results”.

W. B. Carpenter (1813-1885)

> [...] a phenomenon, which, although familiar to every one who takes note of the workings of his own mind, has been scarcely recognised by Metaphysical inquirers ; namely, that when we have been trying to recollect (§ 373) some name, phrase, occurrence, &c., —and, after vainly employing all the expedients we can think-of for bringing the desiderated idea to our minds, have abandoned the attempt as useless,—it will often occur spontaneously a little while afterwards, suddenly flashing (as it were) into our consciousness, either when we are thinking of something altogether different, or on awaking out of profound sleep. (Carpenter, 1875, p. 519)

Preparation: “trying to recollect some ...”, “after vainly employing all the expedients we can think of”, “have abandoned the attempt as useless”.
Incubation: “thinking of something altogether different”, “or on awakening out of profound sleep”.
Illumination: “It will often occur spontaneously a little while afterwards”, “suddenly flashing”.
Verification: -

Frances Power Cobbe (1822-1904)

> It is an every-day occurrence to most of us to forget a particular word, or a line of poetry, and to remember it some hours later, when we have ceased consciously to seek for it. We try, perhaps anxiously, at first to recover it, well aware that it lies somewhere hidden in our memory, but unable to seize it. As the saying is, we 'ransack our brains for it,' but, failing to find it, we at last turn our attention to other matters. By-and-bye, when, so far as consciousness goes, our whole minds are absorbed in a different topic, we exclaim, 'Eureka! the word or verse is so-and-so.' So familiar is this phenomenon, that we are accustomed in similar straits to say, 'Never mind ; I shall think of the missing word by-and-bye, when I am attending to something else'; and we deliberately turn away, not intending finally to abandon the pursuit, but precisely as if we were possessed of an obedient secretary or librarian, whom we could order to hunt up a missing document, or turn out a word in a dictionary, while we amused ourselves with something else. (Carpenter, 1875, p. 519/520, Citing Cobbe)

Preparation: “We try, perhaps anxiously, at first to recover it, well aware that it lies somewhere hidden in our memory, but unable to seize it.”, “'ransack our brains for it,' but, failing to find it,”.
Incubation: “last turn our attention to other matters”, “By-and-bye, when, so far as consciousness goes, our whole minds are absorbed in a different topic”.
Illumination: “we exclaim, 'Eureka!”.
Verification: -

Olivier Wendel Holmes (1809-1894)

> We wish to remember something in the course of conversation. No effort of the will can reach it; but we say, 'Wait a minute, and it will come to me,' and go on talking. Presently, perhaps, some minutes later, the idea we are in search of comes all at once into the mind, delivered like a prepaid parcel laid at the door of consciousness, like a foundling in a basket. How it came there, we know not. The mind must have been at work, groping and feeling for it in the dark; it cannot have come of itself. Yet, all the while, our consciousness, so far as toe are conscious of our consciousness, was busy with other thoughts. (in Carpenter, 1875, p. 520)

Preparation: “We wish to remember something”, “No effort of the will can reach it”.
Incubation: “some minutes later”, “Yet, all the while, our consciousness, so far as toe are conscious of our consciousness, was busy with other thoughts”.

Illumination: “comes all at once into the mind”, “delivered like a prepaid parcel laid at the door of consciousness”.
Verification: -

Sir Benjamin Brodie (1783-1862)

> It seems to me that on some occasions a still more remarkable process takes place in the Mind, which is even more independent of volition than that of which we are speaking; as if there were in the mind a principle of order, which operates without our being at the time conscious of it. It has often happened to me to have been occupied by a particular subject of inquiry; to have accumulated a store of facts connected with it; but to have been able to proceed no further. Then, after an interval of time, without any addition to my stock of knowledge, I have found the obscurity and confusion in which the subject was originally enveloped to have cleared away; the facts have seemed all to settle themselves in their right places, and their mutual relations to have become apparent, although I have not been sensible of having made any distinct effort for that purpose. (in Carpenter, 1875, p. 531)

Preparation: “occupied by a particular subject of inquiry”, “accumulated a store of facts connected with it”, “have been able to proceed no further”.
Incubation: “Then, after an interval of time”.
Illumination: “found the obscurity and confusion ... to have cleared away ”, “the facts have seemed all to settle themselves in their right places”, “their mutual relations to have become apparent”.
Verification: -

William Rowan Hamilton (1805-1865)

> "To-morrow will be the fifteenth birthday of the Quaternions. They started into life, or light, fall-grown, on the 16th of October, 1843, as I was walking with Lady Hamilton to Dublin, and came up to Brougham Bridge. That is to say, I then and there felt the galvanic circuit of thought *close*; and the sparks which fell from it were the *fundamental equations between* i, j, k; *exactly such* as I have used them ever since. I pulled out, on the spot, a pocket-book, which still exists, and made an entry, on which, *at the very moment*, I felt that it might be worth my while to expend the labor of at least ten (or it might be fifteen) years to come. But then it is fair to say that this was because I felt a *problem* to have been at that moment *solved*, — an intellectual *want relieved*, —which had *haunted* me for at least *fifteen years before.*"— (North British Review. Vol. xlv. p. 57.) (Hamilton in Carpenter (1875, p. 537)

Preparation: “an intellectual *want relieved*”, “which had *haunted* me for at least *fifteen years before*”.
Incubation: “as I was walking with Lady Hamilton to Dublin, and came up to Brougham Bridge”.
Illumination: “I then and there felt the galvanic circuit of thought *close*”, “the sparks which fell from it”, “*exactly such*”, “*at the very moment*, I felt that it might be worth my while to expend the labor of at least ten (or it might be fifteen) years to come”, “I felt a *problem* to have been at that moment *solved*”.
Verification: -

Hermann von Helmholtz (1821-1894)

> They often steal into the line of thought without their importance being at first understood; then afterwards some accidental circumstance shows how and under what conditions they have originated; they are present, otherwise, without our knowing whence they came. In other cases they suddenly, without exertion, like an inspiration. As far as my experience goes, they never came at the desk or to a tired brain. I have always so turned my problem about in all directions that I could see in my mind its turns and complications, and run through them freely without writing them down. But to reach that stage was not usually possible without long preliminary work. Then, after the fatigue from this had passed away, an hour of perfect bodily repose and quiet comfort was necessary before the good ideas came. They often came actually in the morning on waking, as expressed in

Goethe's words which I have quoted, and as Gauss also has remarked.[1] But, as I have stated in Heidelberg, they were usually apt to come when comfortably ascending woody hills in sunny weather. The smallest quantity of alcoholic drink seemed to frighten them away. Such moments of fruitful thought were indeed very delightful, but not so the reverse, when the redeeming ideas did not come. For weeks or months I was gnaw-ing at such a question until in my mind I was

> Like to a beast upon a barren heath
>
> Dragged in a circle by an evil spirit,
>
> While all around are pleasant pastures green.

And, lastly, it was often a sharp attack of headache which released me from this strain, and set me free for other interests. (Helmholtz, 1898/1908, p. 284)

Preparation: "But to reach that stage was not usually possible without long preliminary work.", "For weeks or months I was gnaw-ing at such a question".
Incubation: "Then, after the fatigue from this had passed away", "an hour of perfect bodily repose and quiet comfort was necessary", "in the morning on waking".
Illumination: "They often steal into the line of thought", "they are present, otherwise, without our knowing whence they came" "they suddenly, without exertion, like an inspiration".
Verification: -

William W. Ireland (1832-1909)

[...] the mechanical theory - that the whole process is effected without either volition or even any action of the thinking principle, it being merely automatic or mechanical. This opinion he [Carpenter] continues, is unphilosophical, because

It is quite true that after long puzzling ourselves to see the true relations of things, it now and then happens that they suddenly, as it were, present themselves to our mind, and the difficulty is at once solved like a whole landscape seen by a flash of lightning ; but this affords no proof that we have been working at it unconsciously, it merely shows that the mind is sometimes more rapid and powerful in its operations than at others.

Our views are confirmed by considering attentively the nature of these sudden flashes of inspiration; they show us the true relation of things by which we can work out problems and gain results; but they do not give us the results themselves (Ireland, 1875, p. 380-381)

Preparation: "after long puzzling ourselves to see the true relations of things".
Incubation: -
Illumination: "they suddenly, as it were, present themselves to our mind", "the difficulty is at once solved like a whole landscape seen by a flash of lightning", "they show us the true relation of things by which we can work out problems and gain results; but they do not give us the results themselves".
Verification: "they show us the true relation of things by which we can work out problems and gain results; but they do not give us the results themselves"

Friederich August Kekulé (1829-1896)

I frequently, however, spent my evenings with my friend Hugo Müller at Islington at the opposite end of the metropolis. We talked of many things but most often of our beloved chemistry. One fine summer evening I was returning by the last bus, "outside," as usual, through the deserted streets of the city, which are at other times so full of life. I fell into a reverie (Traumerei), and lo, the atoms were gamboling before my eyes! Whenever, hitherto, these diminutive beings had appeared to me, they had always been in motion; but up to that time I had never been able to discern the nature of their motion. Now, however, I saw how, frequently, two smaller atoms

> united to form a pair; how a larger one embraced the two smaller ones; how still larger ones kept hold of three or even four of the smaller; whilst the whole kept whirling in a giddy dance. I saw how the larger ones formed a chain, dragging the smaller ones after them but only at the ends of the chain. I saw what our past master, Kopp, my highly honored teacher and friend, has depicted with such charm in his L'Molekular-welt"; but I saw it long before him. The cry of the conductor: "Clapham Road," awakened me from by dreaming; but I spent a part of the night in putting on paper at least sketches of these dream forms. This was the origin of the "Structural Theory." (Benfey, 1958, p. 22)�.

<u>Preparation</u>: -
<u>Incubation</u>: “I fell into a reverie (Traumerei)”
<u>Illumination</u>: “and lo, the atoms were gamboling before my eyes!”
<u>Verification</u>: “ut I spent a part of the night in putting on paper at least sketches of these dream forms. This was the origin of the 'Structural Theory.'”

> I was sitting writing at my textbook but the work did not progress; my thoughts were elsewhere. I turned my chair to the fire and dozed. Again the atoms were gambolling before my eyes. This time the smaller groups kept modestly in the background. My mental eye, rendered more acute by visions of this kind, could now distinguish larger structures, of manifold conformation; long rows, sometimes more closely fitted together, all twining and twisting in snakelike motion. But look! What was that? One of the snakes had seized hold of its own tail, and the form whirled mockingly before my eyes. As if by a flash of lightning I awoke. (Benfey, 1958, p. 22)�.

<u>Preparation</u>: “but the work did not progress”.
<u>Incubation</u>: “my thoughts were elsewhere”, “I … dozed”.
<u>Illumination</u>: “But look! What was that?”, “As if by a flash of lightning I awoke”.
<u>Verification</u>: -

Child (1892) subjects examples of own experiences to question 1 (p. 455): “ When you are unable to recall the name of something wanted and you say, "Never mind, it will occur to me," are you conscious of any effort of searching after it?”

> This morning I endeavored to recall the name of the characters I had read of in one of Scott's novels the night before. I could remember but one, and then only with much effort. During the morning I was unable to recall any other character by name, although constantly endeavoring to do so. After teaching a Sunday- school class, I walked home in the afternoon with my mother, and, without any effort, gave not only the names of the principal characters but many of the unimportant. I had not thought of the work for a number of hours. (Child, 1892)

<u>Preparation</u>: “endavored to recall” “much effort” “unable to recall […] although constantly endeavoring to do so”
<u>Incubation</u>: “I walked home in the afternoon”, “I had not thought of the work for a number of hours”.
<u>Illumination</u>: “without any effort, gave”
<u>Verification</u>: -

> I was trying to think of the name of a book, and gave it up. About half an hour after, I was talking of something else when all of a sudden, I blurted out the name without any conscious volition on my part, or without thinking anything about the book at all.

<u>Preparation</u>: “I was trying to think of”, “and gave it up”.
<u>Incubation</u>: “I was talking of something else”, “without any conscious volition on my part”.
<u>Illumination</u>: “when all of a sudden”.
<u>Verification</u>: -

> I have tried to think of the name of a person without success in the evening, and the next morning have had it come to me without any connecting ideas at all, but it just seemed to 'pop' into my mind.

Preparation: “I have tried to think of”, “without success”.
Incubation: “in the evening” … “the next morning”.
Illumination: “had it come to me”, “it just seemed to 'pop' into my mind”.
Verification: -

> I was telling my sister of a young lady, but I could not remember her name, though I thought I knew it. At last I had to give it up, and after a while forgot all about it, though I could not at first force myself to think entirely of other things. For a time I was dimly conscious of trying to remember. The next morning the name suddenly flashed across my mind, apparently without being suggested by anything else.

Preparation: “I could not remember her name”, “At last I had to give it up”
Incubation: “after a while forgot all about it”
Illumination: “next morning the name suddenly flashed across my mind”
Verification: -

Child 1890 Question III “When perplexed at your progress in any work (mathematical, professional, literary, chess, puzzles, etc) have you ever left it unfinished and turned your attention to other things, and after some time, on voluntarily returning to it, have found yourself able at once to satisfactorily master it? ”

> Often while playing chess or working an example I have not succeeded well. On returning after having left it for a while, what was difficult before seemed now very easy.

Preparation: “Often while playing chess or working an example I have not succeeded well”
Incubation: “On returning after having left it for a while”
Illumination: “what was difficult before seemed now very easy”
Verification: -

> In working mathematical examples in the evening I some- times 'get stuck.' I leave it over night and take it up in the morning and I often get the answer immediately. So in translation I find passages that I cannot get out. I study on them for a while and then leave them for several hours, or better sometimes days, and I can get them clearly.

Preparation: “working mathematical examples in the evening I some- times 'get stuck.'” “passages that I cannot get out” “I study on them for a while”
Incubation: “I leave it over night” “then leave them for several hours, or better sometimes days”
Illumination: “I often get the answer immediately” “I can get them clearly.”
Verification: “take it up in the morning”

> In writing music I often get to a stumbling-block, and try vainly to search for a chord or bar of music, but cannot find the thing I want. When it gets me very excited I leave it and go for a walk, and on coming back to work, I will most likely be able to write it out at once, seemingly without any work on my part; it is all ready for me to put down. I have frequently had the experience.

Preparation: “often get to a stumbling-block”, “try vainly to search for”, “but cannot find the thing I want”, “ it gets me very excited”.
Incubation: “I leave it and go for a walk”
Illumination: -
Verification: “I will most likely be able to write it out at once”.

> I have come across a sentence that was particularly difficult in some Latin book I was reading, and have been unable to translate it. I have then turned my attention to abstruse problems in mathematics, and worked for some time. On returning to the Latin I have often found it quite simple, and have sometimes translated it at sight.

Preparation: "a sentence that was particularly difficult", "unable to"
Incubation: "then turned my attention"
Illumination: -
Verification: "I have often found it quite simple, and have sometimes translated it at sight."

Question IV (p. 458): "During sleep have you ever pursued a logical, connected train of thought, upon some topic or problem, in which you have reached some conclusion, and the steps and conclusion of which you have remembered on awakening?"

> I have played a game of chess in my sleep. The game seemed in my sleep to be entirely completed. In the morning I remembered all but one or two plays, and when I played the game over in the morning it seemed consistent. I do not think that I had ever played that game (i. e., a game with those identical moves) before, and I could not play it now. I had been playing a great deal at the time though, and of course had been thinking of chess when I went to bed.

Preparation: "I had been playing a great deal at the time though, and of course had been thinking of chess when I went to bed."
Incubation: "I have played a game of chess in my sleep."
Illumination: -
Verification: "when I played the game over in the morning it seemed consistent"

> I have been puzzled by a problem in algebra which I found it impossible to solve and let it rest over night, and while asleep have thought out each step and remembered it, and in the morning on trying the problem again, solved it without difficulty.

Preparation: "puzzled by a problem in algebra which I found it impossible to solve"
Incubation: "let it rest over night" "while asleep have thought out each step"
Illumination: -
Verification: "and remembered it", "in the morning on trying the problem again, solved it without difficulty"

> Being greatly troubled over a problem in algebra just before going to sleep, and leaving the problem half finished, I dreamed the rest of the solution and obtained the correct result. On awaking, I remembered it, and it was correct.

Preparation: "Being greatly troubled over a problem", "leaving the problem half finished"
Incubation: "I dreamed the rest of the solution"
Illumination: -
Verification: "I remembered it, and it was correct."

> In my senior year at college I had an essay to write that troubled me unusually. After trying to decide upon the subject until quite late, I fell asleep and dreamed not only of the subject and analysis but of all the details. The

> next morning I wrote out what I had dreamed, and found it far more satisfactory than anything I had ever done in the same line before. Two years before I had exactly the same experience about an equation in algebra which I worked out correctly in sleep.

Preparation: “that troubled me unusually”.
Incubation: “I fell asleep and dreamed not only of the subject and analysis but of all the details”.
Illumination: -
Verification: “next morning I wrote out what I had dreamed”, “and found it far more satisfactory”.

Question V (p. 460) “Have you ever been conscious of having discovered something new, e.g., an invention, a literary or poetical creation, or a mathematical solution etc.? ”

> I can instance as frequent the smallest kind of literary creation, forms of verbal expression, what one may call an apt phrase coming to my mind suddenly, uncalled for, as if uttered by some one else, of no use to me at the time or perhaps ever.

Preparation: -
Incubation: “uncalled for”, “as if uttered by some one else”.
Illumination: “apt phrase coming to my mind suddenly, uncalled for”.
Verification: -

> Many instances of mathematical or psychological problems have suddenly flashed across my mind when on a totally different subject; sometimes very distinct and sometimes indistinct, which I afterwards developed into distinctness.

Preparation: -
Incubation: “when on a totally different subject”, “sometimes very distinct and sometimes indistinct”.
Illumination: “suddenly flashed across my mind”.
Verification: “which I afterwards developed into distinctness”.

> Productive thinking involves going from a situation of bewilderment or confusion about some issue that is blind to the core structural features and properties of that issue, to a new state in which everything about the issue is clear, makes sense, and fits together. (Wertheimer, 1996, p. 4/5).

Preparation: “situation of bewilderment” or “confusion”, “blind to the core structural features”.
Incubation: -
Illumination:“is clear” “makes sense” “fits together”
Verification: -

> In the first page of his introduction, Wertheimer called it "the transition from a blind attitude to understanding in a productive process," a "surprising event," "the birth of a genuine idea," "when one has begun really to grasp an issue." At the core of the process is a kind of reorganization or restructuring, going from a state that makes no sense to one that does make sense, displays understanding or insight, is crystal clear. It is a transition from a state that is meaningless, nonsensical, befuddled to one that is meaningful -- a "good Gestalt." (Wertheimer (1996, p. 5)

Preparation: "the transition from a blind attitude to understanding in a productive process,"
Incubation: -

Illumination: "surprising event", “It is a transition from a state that is meaningless, nonsensical, befuddled to one that is meaningful”
Verification: -

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