

1 *Nonlinear Dynamics, Psychology, and Life Sciences, Vol. 13, No. 1, pp. 75-94.*
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4 **1/f Scaling in Movement Time Changes with Practice in** 5 **Precision Aiming**

6

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12 ***Abstract:** When people perform repeated goal-directed movements, consecutive*
13 *movement durations inevitably vary over trials, in poor as well as in skilled*
14 *performances. The well-established paradigm of precision-aiming is taken as a*
15 *methodological framework here. Evidence is provided that movement variability*
16 *in closed tasks is not a random phenomenon, but rather shows a coherent*
17 *temporal structure, referred to as 1/f scaling. The scaling relation appears more*
18 *clearly as participants become trained in a highly constrained motor task. Also*
19 *Recurrence Quantification Analysis (RQA) and Sample Entropy (SampEn) as*
20 *analytic tools show that variation of movement times becomes less random and*
21 *more patterned with motor learning. This suggests that motor learning can be*
22 *regarded as an emergent, dynamical fusing of collaborating subsystems into a*
23 *lower-dimensional organization. These results support the idea that 1/f scaling*
24 *is ubiquitous throughout the cognitive system, and suggest that it plays a*
25 *fundamental role in the coordination of cognitive as well as motor function.*

26 **Key Words:** fractal scaling relations, nonlinear dynamics, motor coordination,
27 degrees-of-freedom, task complexity

28

INTRODUCTION

29 Repeated instances of human performance are usually measured using
30 summary statistics of central tendency and average variation around a central
31 tendency. It can be more informative however to complement summary
32 measures with time-evolutionary measurements (Riley & Turvey, 2002; Slifkin
33 & Newell, 1999). Time series of measured values can be qualitatively different
34 for identical means and standard deviations. For example, consider an artificial

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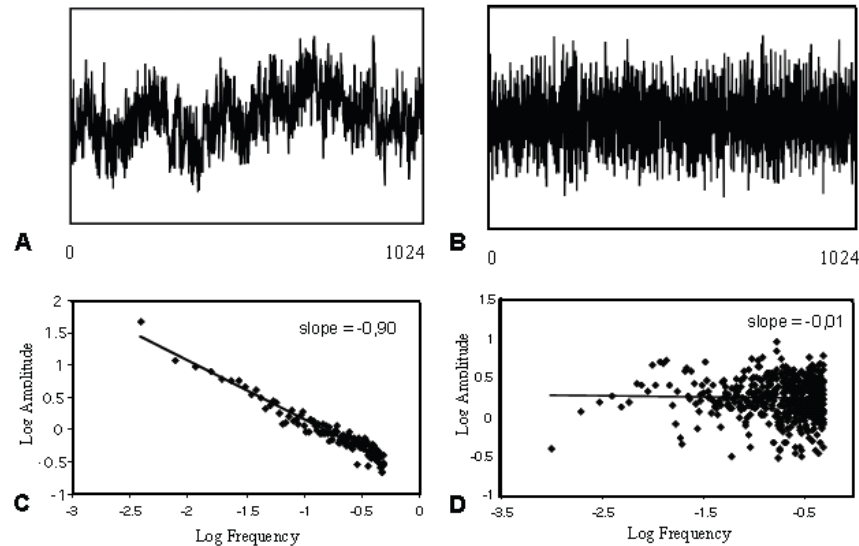
35 time series in which measured values follow an idealized sine wave across the
36 trials of an experiment; measurements fluctuate around the mean in a
37 deterministic, non-random cycle. Compare that with the same “sine wave” data
38 rearranged in a random sequence of occurrence. The respective time series have
39 equivalent means and standard deviations, but one comes from a random process
40 and the other from a simple oscillating process.

41 Repeated measures of human performance oscillate in a more complex
42 pattern than the sine wave, but it is a pattern nonetheless, and may prove just as
43 revealing of underlying dynamics. Especially helpful in this regard are recent
44 advances in the study of nonlinear dynamics. By applying an advanced
45 nonlinear toolbox, it is possible to gauge fractal patterns in data, as well as
46 indices of determinism or entropy and other descriptor variables (Riley,
47 Balasubramaniam, & Turvey, 1999; Slifkin & Newell 1999). These tools are
48 applied in the present case to test whether the pattern of variation changes with
49 practice in a simple perception-action task. Our starting point is the observation
50 of $1/f$ scaling in time series of human performance – the widely observed finding
51 of long range correlations across successive data points in motor coordination
52 experiments (Riley & Turvey, 2002; Slifkin & Newell, 1999; Treffner & Kelso,
53 1999) and cognitive performances (Gilden, Thornton, & Mallon, 1995; Gilden,
54 2001; Van Orden, Holden, & Turvey, 2003).

55 The widely observed $1/f$ scaling relation expresses aperiodic, fractal
56 fluctuations of available frequencies across a time series of data. In a spectral
57 decomposition of the data signal, however, the amplitude at a particular
58 frequency of fluctuation is inversely proportional to the frequency itself. One
59 observes a nonlinear, log-log relation between the frequency of variation across
60 the data series and the magnitude of variation, for a given data set.

61 The pattern implies that no characteristic scales dominate the
62 underlying process; the same dynamics occur at every scale, including very high
63 amplitude and low frequency fluctuations. In fact, the more data one collects –
64 that is the longer the data series – the larger the magnitude of variation for the
65 whole set (Van Orden, Holden, & Turvey, 2005). Consequently the implicit
66 amount of variance is undefined as total explicit variability increases rather than
67 stabilizes when larger samples are collected (Gilden, 2001; Holden, 2005;
68 Mandelbrot, 1982). Interestingly, $1/f$ scaling appears to be a ubiquitous property
69 of repeated measures in human performance (Kello, Beltz, Holden, & Van
70 Orden, 2007). An example data series yielding a $1/f$ scaling pattern is presented
71 in Fig. 1a.

72 The phenomenon of $1/f$ scaling demonstrates the importance of
73 considering how variability scales with sample size in behavioral data (Riley &
74 Turvey, 2002). This information is not implied by the sampled amount of
75 variability and can only be obtained by incorporating the dynamical properties
76 of behavioral data as an essential aspect of measurement. Time series
77 phenomena like $1/f$ scaling are simply unavailable in summary statistics such as
78 central tendency or magnitude of variation. As in the example of the sine wave,
79 $1/f$ scaling disappears if the original order of measurement is randomized.



80
 81 **Fig. 1.** A typical example of $1/f$ scaling in an intact behavioral time series of one
 82 participant (a), and the same time series after randomization (b), and their
 83 respective power spectra (c and d). A slope of -1 indicates ideal $1/f$ scaling, a
 84 slope of 0 indicates random sequential ordering, see Method section.

85 Figure 1 illustrates this point using actual data. Figure 1b shows the
 86 same data series presented in Fig. 1a after randomizing the sequence trial order
 87 in which the data points were collected. The same mean and standard deviation
 88 are computed from the randomized time series, but the time-evolutionary scaling
 89 relation is erased (compare spectra in Fig. 1c and Fig. 1d). The rationale for
 90 summary statistics, however, the central limit theorem, specifies that collective
 91 aggregate properties of *independent* components obey a Gaussian distribution.
 92 Consequently, measured over a duration or sample size T , the standard deviation
 93 of a data series will increase as T^h where the exponent $h = 1/2$ implies randomness.
 94 For fractal processes like $1/f$ scaling, however, h exceeds that value, which calls
 95 into question the basic justification of the summary statistics (Mandelbrot,
 96 1982).

97 **Changing Dynamics with Motor Learning**

98 Although the occurrence of $1/f$ scaling is widely reported, the
 99 underlying mechanism remains an enigma throughout the physical, biological,
 100 and psychological sciences. Apart from its presence, tempting issues remain
 101 such as why the relative presence of $1/f$ scaling changes in different human
 102 performances. Whereas decreasing amounts of variability typically indicate
 103 improving levels of performance (e.g. Fitts, 1954), no such general statement

104 can be made with respect to the temporal structure of variability in human
105 performance. An important suggestion, however, is that the structure of
106 movement variability may provide important clues regarding the compression of
107 degrees of freedom into a controllable, low-dimensional coordinative structure
108 (Mitra, Amazeen, & Turvey, 1998; Riley & Turvey, 2002; Turvey, 1990). In this
109 article we pursue consequences of this suggestion.

110 The specific question of the present research is whether fractal patterns
111 change after practice in precision aiming. Pointing or precision aiming is a long-
112 established paradigm to study coordination of perception and action. In
113 precision aiming, participants might move a pointer or a computer mouse
114 between designated targets. In our experiment they move a stylus back and
115 forth, repeatedly, between two targets on a digital tablet. In general, targets can
116 be wide or narrow in diameter and closer or further apart, both of which affect
117 performance. Fitts' law takes into account target width and the distance between
118 targets to accurately predict movement-time central tendency, given accuracy
119 greater than 96% (Fitts, 1954). The study that we report in this article used
120 conditions yielding performance well below the 96% accuracy criterion. The
121 purpose was to gauge changes in performance after motor practice in precision
122 aiming. To further insure the opportunity for performance to improve, we
123 required non-dominant hand performance.

124 Our specific interest is change in the structure of variation in movement
125 times. This interest stems from recent developments in complexity theory and
126 widespread observations of complex variation in perception-action tasks. Yet it
127 remains to be discovered whether the structure of variation changes due to
128 training in perception-action tasks.

129 We assume that $1/f$ scaling is a reflection of intrinsic self-organizing
130 interaction-dominant dynamics (Van Orden et al., 2003). If so, then the logic of
131 our experiment follows: first, $1/f$ scaling should be observed in movement time
132 series of precision-aiming performance, as the phenomenon is claimed to be
133 universal. Second, measured values of poor performance reflect less stable, less
134 systematic coordination of perception and action. Third, instabilities contribute
135 unsystematic perturbations to measured values. Fourth, unsystematic
136 perturbations add random variation to the signal of $1/f$ scaling as white noise.
137 Fifth, each participant's time series should show reduced effects of random
138 variation after practice, and more clear signals of $1/f$ scaling.

139 By using small targets, relatively far apart, and requiring the use of the
140 non-dominant hand we induce less stable, less systematic coordination of
141 perception and action. Because these conditions induce relatively poor
142 performance overall, they also allow plenty of room for improvement with
143 practice. The assertion is that improvement comes about by compressing the
144 available degrees of freedom. Unfortunately inducing very poor performance
145 overall reduces the possibility of reliably estimating directly the active degrees
146 of freedom.

147 For instance, in the framework outlined by Mitra et al. (1998) we must
148 expect to deal with the early phase of motor learning in which the system

149 discovers and establishes the relevant collective variable. As they explain, in this
150 phase there may be competing collective variables and candidate subsystems at
151 the level of the coordination pattern. In contrast, intermediate phases refine the
152 interactions among subsystems that contribute to the victorious collective
153 variable. Nevertheless, both early and intermediate phases of motor learning
154 reduce active degrees of freedom, which we may discover indirectly in fractal,
155 recurrence quantification, and sample entropy analyses.

156 As participants improve performance of the precision aiming task, we
157 predict clearer examples of $1/f$ scaling in the movement time series. The
158 rationale is that in learning, the many degrees of freedom for movement, that is,
159 the available possibilities for the body to move between targets in precision
160 aiming, are reduced to promote more efficient and coordinated performance
161 (Bernstein, 1967). Movement will not be organized randomly, a situation in
162 which all (indeterminate) degrees of freedom would be available. And
163 movements will not be overly persistent (as in the sine wave), since contextual
164 constraints on the kinematics of forthcoming movements are always
165 dynamically changing. Apparent $1/f$ scaling is situated on the hypothetical
166 border between persistence and “random” (chaotic) variability, between order
167 and disorder. So, clearer instances of $1/f$ scaling should be observed with
168 decreasing available degrees of freedom, as performance more reliably gauges
169 variation near the border between order and chaos.

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METHOD

Participants

172 The participants were fifteen undergraduate students who received
173 course credit for participation. None suffered from any known motor
174 impairment and all participants had normal or corrected to normal vision. All
175 participants were right-handed as tested by the handedness subscale of the
176 Lateral Preference Inventory (Coren, 1993).

177

Materials

178 Movement coordinates were recorded using a WACOM digitizer tablet
179 connected to a regular Pentium PC. The tablet samples at temporal rate of
180 171Hz, with a spatial resolution of 1000 lines/cm. The input device was an
181 inkless stylus used on a model sheet (A4) placed on top of the digitizer tablet.
182 Kinematic records were converted into two dimensional coordinates using Oasis
183 software (De Jong, Hulstijn, Kosterman, & Smits-Engelsman, 1996). Particip-
184 ants were seated on a height-adjustable chair in front of the digitizer tablet.

185

Procedure

186 In the present study, participants were invited to draw lines back and
187 forth between two visual targets, as fast and as accurately as possible. The
188 targets were presented on a printed sheet of paper, one at the left side of the
189 paper and one at the right side. Participants were allowed to modify the distance

190 to the digitizer tablet and the digitizer's orientation within a deviating range of
191 30° from the central position. The target width was 0.4 cm and the distance
192 between targets was 24 cm. Five blocks of 1100 trials were completed with the
193 non-dominant hand, all separated by three-minute breaks. When the last trial in a
194 block was reached, a tone signaled the end of the block.

195

Analyses

196 Movement times between targets were treated as a time series. To
197 quantify the temporal structure of the successive fluctuations, Spectral Analysis,
198 Standardized Dispersion Analysis (SDA), and Detrended-Fluctuation Analysis
199 (DFA) were conducted. To further investigate those results we fit the $1/f$ + white
200 noise model of Thornton and Gildea (2005), conducted a Recurrence
201 Quantification Analysis (RQA), and tested for sample entropy (SampEn). All
202 analyses were performed using Matlab scripts.

203 Human time series data, like data from biological systems generally,
204 are typically non-stationary noisy series containing extreme values. The tools
205 available for fractal analyses must work around problems that come with such
206 data. Known problems can be compensated for, which is why we used several
207 methods together to estimate change across fractal statistics of practice blocks.

208 Some methods are complementary in that the strengths of each
209 compensate for the weaknesses of the others. For instance, spectral analysis,
210 while robust in many respects, requires extensive preprocessing of the signal and
211 extreme observations can contaminate the outcome of the analysis (see Holden,
212 2005; Press et. al, 1992). Nonetheless they give a clear picture of $1/f$ scaling in
213 the low frequency region of the spectral plot. Detrended fluctuation analysis is
214 reliable and robust, and does not require the arbitrary setting of parameters, as
215 does spectral analysis (Eke et al., 2002). Detrended fluctuation analysis can be
216 applied to nonstationary signals and is not susceptible to most statistical artifacts
217 or long-term trends, but it can falsely classify certain types of signals as fractal
218 (Rangarajan & Ding, 2000). Standardized dispersion analysis is also highly
219 reliable, but linear and quadratic trends may bias its output (we therefore remove
220 both linear and quadratic trends for SDA). We insure reliable conclusions by
221 using all three methods together.

222 An important advantage of RQA, unlike the aforementioned methods,
223 is that this technique does not impose constraints on data set size. RQA does not
224 make assumptions regarding statistical distributions or stationarity of data either.
225 The challenge of applying RQA measures specifically as a complementary tool
226 for fractal analyses is addressed in this paper.

227 Spectral Analysis

228 Spectral analysis transforms data series from the time domain
229 (milliseconds) into a frequency domain (Hz), through a Fast-Fourier
230 Transformation. The procedure finds the best-fitting sum of sine and cosine
231 waves in a data signal, and renders their amplitudes and frequencies on log-log

232 scales. The statistic of interest is the slope of the spectral portrait, which
233 captures the relation between amplitudes and frequencies of variation in the data
234 signal. A zero slope indicates non-random random structure in the signal, a slope
235 of -1 indicates $1/f$ scaling. Spectral slopes as steep as -2 indicate fractional
236 Brownian motion, the epitome of random walk processes.

237 Spectral analysis requires some preprocessing of the raw data (Holden,
238 2005). Extreme values were excluded (values below 50 ms and above 850 ms in
239 the present case). Next, remaining outliers were removed if they lay outside a 3
240 x SD criterion. Finally, linear trends were removed and the remaining data were
241 truncated to 1024 trials. The number of estimated frequencies was 512, and the
242 spectral slopes were calculated over the 25% of lowest frequencies.

243 **Standardized Dispersion Analysis (SDA)**

244 Dispersion analysis assesses the relative coherence of the patterns of
245 fluctuations in $1/f$ scaling via the fractal-dimension statistic (see Holden, 2005).
246 The Fractal Dimension (FD) is derived from estimating how variability changes
247 with changing sample sizes. The dispersion analysis describes the changes in the
248 variability of a measurement across a range of sample sizes (or measurement
249 resolutions), in terms of a power-law scaling relation. In other words, the
250 dispersion analysis determines a scaling relation between sample size and
251 sample variability. This relation is estimated in the slope of a regression line
252 across successive estimates of how variability changes with sample size, in this
253 case across six estimates. An FD of 1.5 indicates a random data series, whereas
254 values approaching 1.20 indicate $1/f$ scaling.

255 **Detrended Fluctuation Analysis (DFA)**

256 Detrended-fluctuation analysis (Peng et al., 1993) represents a relation
257 between window sizes of data and the mean standard-deviations of the
258 windowed data. First, the time series is subdivided into non-overlapping bins of
259 equal length, and in each bin, the local trend -the locally best-fit line- is
260 subtracted. Next, the root-mean-square of the locally detrended and binned
261 timeseries is computed for windows of the same length. The process is repeated
262 over increasing window sizes out to the limits of the finite data set. In the
263 present study, DFA was performed on window sizes ranging between 4 and
264 1024. When the average fluctuation is plotted over the increasing window sizes
265 on log-log scales, the slope represents the $1/f$ scaling exponent. A resulting
266 scaling exponent equal to 0.5 would correspond to white noise. If the scaling
267 exponent exceeds 0.5, the series has long range persistent correlations. In the
268 case of a scaling exponent equal to 1, the sequence is scaled exactly as $1/f$.

269 **The $1/f$ + White Noise Model**

270 The model proposed by Thornton and Gilden (2005) assigns data series
271 the likelihood they originate from a fractal as opposed to Auto-Regressive
272 Moving-Average (ARMA) process (cf. Wagenmakers, Farrell, & Ratcliff, 2004).

273 This likelihood is based upon the comparison of a data set against model fitting
274 parameters for whitened fractal noise (a mixture of $1/f$ scaling and Gaussian
275 noise) as well as ARMA processes. These fitting parameters are given in
276 separate reference libraries based on the 800 sampling distributions generated by
277 the two candidate processes. The libraries encapsulate a reasonably complete
278 range of spectral shapes that may be observed in either of the models. Based on
279 maximum likelihood, the libraries are used to find the most likely source of an
280 input data spectrum. Through this procedure, the classifier is able to decide
281 whether a given data set is more consistent with a fractal or an ARMA
282 interpretation. When this spectral classification framework favors a fractal
283 interpretation, a $1/f +$ Gaussian noise model is tested. An advantage of this
284 technique is that no prior assumptions are made concerning the nature of the
285 data. In the present case, the $1/f +$ Gaussian noise model was generally preferred,
286 and thus constitutes another test to determine changes due to practice. In
287 particular, this model returns a specific test of whether white noise amplitude
288 decreases due to practice.

289 **Recurrence Quantification Analysis (RQA)**

290 RQA combines recurrence plots (Eckmann, Kamphorst, & Ruelle,
291 1987), that is, the visualization of trajectories in phase space, with the objective
292 quantification of (nonlinear) system properties. That is, time series are delayed
293 with a certain lag (Takens, 1981) and embedded in a phase space with an
294 appropriate dimensionality. Subsequently, complexity measures are quantified
295 in that reconstructed phase space. This technique reveals subtle time-
296 evolutionary behavior of complex systems by quantifying system characteristics
297 in reconstructed phase-space.

298 RQA measures include recurrence (the percentage of data points that
299 share a common area in phase space, dependent on a defined radius - the mean
300 Euclidean distance separating data points in reconstructed phase space),
301 determinism (the percentage of recurrent points that constitute line segments -
302 recurrent patterns- parallel to the diagonal identity line in a recurrence plot),
303 entropy (the Shannon entropy of the distribution of deterministic line segments.
304 The index is one way to quantify complexity of a deterministic structure),
305 maxline (a measure of dynamical stability inversely proportional to the largest
306 positive Lyapunov exponent, hence, attractor strength), and trend (the degree of
307 nonstationarity). Detailed tutorials that include a careful examination of these
308 parameters are (Marwan, Romano, Thiel, & Kurths, 2007; Riley,
309 Balasubramaniam, & Turvey, 1999; Riley & Van Orden, 2005).

310 Parameters that affect the outcome of RQA measures, and thus need to
311 be chosen carefully, are time lag or delay, and the embedding dimension. Here a
312 delay of 3 was combined with an embedding dimension of 4. These choices
313 were based on the first local minimum of the Average Mutual Information
314 function (Fraser & Swinney, 1986) for the delay, and global False Nearest
315 Neighbors (Kennel, Brown, & Abarbanel, 1992) for the embedding dimension.

316 Another parameter is the minimal line length for identifying deterministic
317 segments; here it was set to two points.

318 We applied a different RQA strategy than the one that typically is
319 chosen. Traditionally, recurrence is identified by choosing first a fixed radius.
320 We reversed that order, so that our a priori choice was the level of recurrence,
321 not the radius. Instead of a fixed radius we used a fixed amount of recurrence
322 (5%), and the resultant radius, for each participant, was the dependent variable.
323 When a smaller radius is observed for the same level of recurrence, it implies
324 that the absolute level of recurrence is higher.

325 **Sample Entropy**

326 Entropy measures have previously been used as an indirect gauge of the
327 dynamical degrees-of-freedom in complex data signals (e.g. Newell, Broderick,
328 Deutsch, & Slifkin, 2003; Slifkin & Newell, 1999). To compare the direction of
329 change of the various indices of dynamical degrees-of-freedom described in the
330 previous sections, sample entropy was computed (Richman & Moorman, 2000).

331 The Sample Entropy (SampEn) index indicates whether the
332 dimensionality of the reconstructed attractor is increasing or decreasing.
333 $\text{SampEn}(m,r,N)$ is precisely the negative natural logarithm of the conditional
334 probability that a dataset of length N , having repeated itself within a tolerance r
335 for m points, will also repeat itself for $m + 1$ points, without allowing self-
336 matches. SampEn measures generally range between 0 and 2; more random data
337 sets produce a higher entropy value, and more regular data are reflected by
338 lower values.

339 In the present SampEn analysis, we used parameter values of $m = 3$ and
340 filter width of $r = 0.1$, where m is the length of compared runs of data and r is
341 the proportion of the standard deviation used to filter the data; a detailed outline
342 of the procedures for calculating SampleEn and determining its parameter values
343 can be found in Richman and Moorman (2000). Sample entropy has the
344 advantage over approximate entropy because it is less biased (i.e., SampEn does
345 not include self-matches), and more robust over a range of input parameters
346 (Lake, Richman, Griffin, & Moorman, 2002). The sample entropy, which is
347 computed over the sequential values of the time series, should not be confused
348 with the entropy in RQA, which is measured over the distribution of
349 deterministic line segments in the recurrence plot.

350

RESULTS

351 The discussion of the results starts with a summary of the traditional
352 performance measures. These analyses pertain to successive movement times,
353 their standard deviations, accuracy levels, and their changes with practice. Then,
354 the results from the spectral and fractal analyses are presented, followed by the
355 outcome of fitting the $1/f$ + white noise model. Then, the RQA outcomes are
356 presented.

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Performance measures

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The overall mean movement time was 590 ms (\pm 80 ms). Not surprisingly, a repeated measures ANOVA across the 5 blocks of practice found decreasing mean movement times and standard deviations with practice (block: 1 (625 ms, $SD = .09$) vs. 2 (620 ms, $SD = .08$) vs. 3 (606 ms, $SD = .08$) vs. 4 (556 ms, $SD = .08$) vs. 5 (542 ms, $SD = .07$), very near the threshold for statistical significance ($F(1, 14) = 4.51, p < .06$ and $F(1, 14) = 2.83, p < .06$ respectively); see Fig. 2a. To further investigate these changes, difference contrasts were computed. For the movement times, the change between block 3 and block 4 was statistically significant, $F(1,14) = 6.74, p < .05$. The movement times decreased even more in block 5, $F(1,14) = 5.70, p < .05$. The difference contrasts between the other blocks were not statistically significant.

Each practice block was divided in four non-overlapping epochs of 256 data points to investigate possible changes in movement times within each block. Within the first and the fourth block, movement times decreased significantly between subsequent epochs, $F(3,42) = 6.74, p < .01$ and $F(3,42) = 5.95, p < .01$ respectively. Throughout the other blocks, the repeated measures ANOVAs were not significant. However, a careful examination of the data revealed that the difference contrasts between epoch 1 and 2 showed an initial drop in movement time (block 2: $F(1,14) = 4.82, p < .05$; block 3: $F(1,14) = 15.11, p < .01$; block 5: $F(1,14) = 5.95, p < .05$), after which movement times stabilized for the remainder of that block. Practice block did not reliably affect accuracy (block: 1 (15.37%, $SD = 10.25$) vs. 2 (14.40%, $SD = 10.26$) vs. 3 (15.23%, $SD = 8.11$) vs. 4 (13.93%, $SD = 7.77$) vs. 5 (12.13%, $SD = 9.3$), all $F_s < 1$).

Spectral and Fractal Analyses

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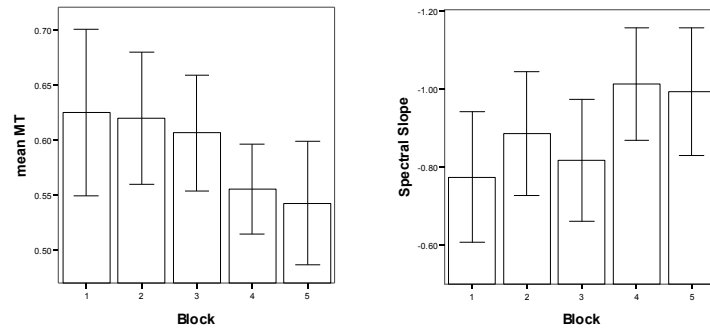
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The outcomes of spectral analyses, standardized dispersion analyses (SDA), and detrended fluctuation analyses (DFA), were subjected to repeated measures ANOVAs, to test for changes in scaling across blocks of practice. The spectral analyses all yielded slopes consistent with $1/f$ scaling, with average scaling exponents less than or equal to negative one. The main effect of block was significant ($F(4, 56) = 4.65, p < .01$), revealing a significant linear trend with decreasing scaling exponents across practice blocks (the spectral slopes become steeper with practice), $F(1, 14) = 11.07, p < .01$. This pattern was confirmed by the SDA ($F(4, 56) = 3.55, p < .01$), revealing a significant linear trend with decreasing fractal dimensions, $F(1, 14) = 9.74, p < .01$. Likewise the DFA revealed clearer examples of $1/f$ scaling with practice; over blocks, $F(4, 56) = 2.63, p < .05$, and a significant linear trend with increasing scaling exponents, $F(1, 14) = 4.48, p < .05$.

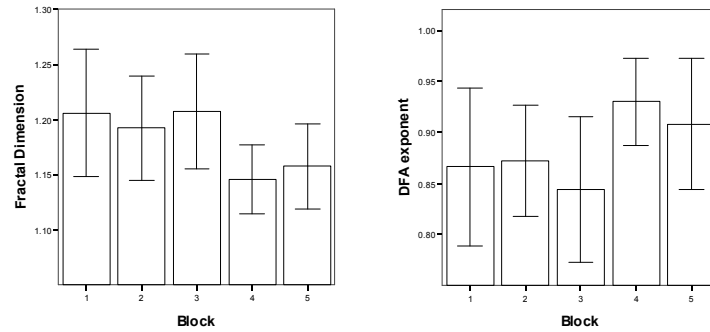
To further investigate these effects, the mean difference contrasts between blocks were examined. Only the third and the fourth practice blocks differed reliably. For the spectral analysis, SDA and DFA, $F(1, 14) = 13.39, p < .01$; $F(1, 14) = 10.35, p < .01$; and $F(1, 14) = 6.73, p < .05$, respectively. Other blocks did not differ reliably from temporally adjacent blocks. The

400 changes in the outcome of the spectral analysis, SDA and DFA are illustrated in
 401 Figs. 2b, 2c and 2d respectively. Over blocks, the temporal variation in
 402 movement times became more clearly patterned as a 1/f signal.

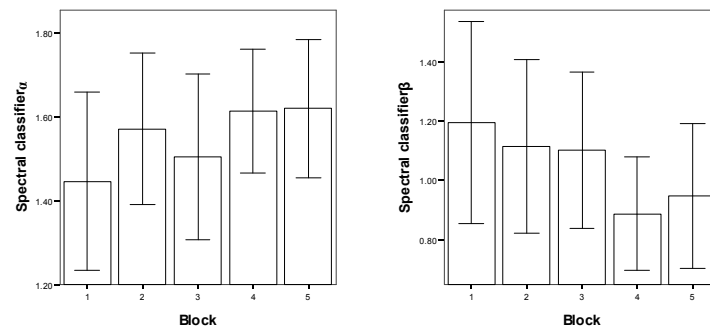
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Fig. 2. Changes in (a) movement time (b) spectral scaling exponent (c) fractal dimension, (d) DFA scaling exponent, (e) scaling exponent α and (f) error term β from Thornton & Gilden's (2005) fBmW model across blocks of practice.

409 To further investigate changes in scaling, within-block changes were
410 estimated by subdividing the movement time series in four non-overlapping
411 epochs of 256 trials. Delignières et al. (2006) showed that for simulated data
412 series, reasonably reliable scaling estimates can be derived from a data series
413 containing 256 trials. However, scaling outcomes over such short time frames
414 are more variable than outcomes over longer time frames. Within block 1, block
415 4 and block 5, none of the scaling estimates changed reliably, all F 's < 1 . In
416 blocks 2 and 3, the different scaling estimates did not converge, likely because
417 short time series are bound to reveal more variable indices. Within block 2, only
418 SDA showed higher FD's (becoming less like ideal $1/f$ scaling) across epochs,
419 $F(3,42) = 3.50, p < .05$. Throughout block 3, spectral exponents did increase
420 (becoming more like ideal $1/f$ scaling) and the DFA exponents decreased (also
421 becoming more like ideal $1/f$ scaling), $F(3,42) = 3.15, p < .05$ and $F(3,42) = 9.43,$
422 $p < .001$ respectively.

423

The $1/f$ + White Noise Model

424 The spectral classification framework assigned a larger likelihood to the
425 $1/f$ + white noise model for 82.7 % of the time series as opposed to an ARMA-
426 model, $t(148) = -3.50, p < .01$. Thus, changes due to practice were only
427 examined using fits to the $1/f$ + white noise model. Time series were first
428 standardized and then transformed into an 8-point composite spectrum, averaged
429 over participants, a procedure described by Thornton and Gilden (2005). The
430 application of Thornton and Gilden's model showed a direction of change that
431 was consistent with the other fractal scaling estimates. Although the spectral
432 exponents suggested more pronounced fractal scaling after more blocks of
433 practice, that increase was not statistically significant, $F(4, 56) = 1.363, p = .25$.
434 The random error term, however, did reliably decrease with blocks of practice,
435 $F(4,56) = 2.99, p < .05$, as a statistically significant linear trend over practice
436 blocks, $F(1,14) = 5.25, p < .05$. This outcome is relatively direct support that
437 random sources of variation decrease with practice, better revealing a $1/f$ signal.
438 These outcomes are illustrated in Figs. 2e and 2f.

439

Recurrence Quantification Analysis

440 RQA was performed to examine time-evolutionary properties of the
441 time series that cannot be detected using scaling measures. Univariate repeated
442 measures ANOVAs did not reveal significant changes in radius with practice for
443 the intact data ($F(4,56) = 1.60, p < .19$). (However, the difference contrast
444 between block 3 and 4 was close to statistical significance, $F(1,14) = 3.74, p$
445 $< .08$). Also trend did not change over practice blocks, $F < 1$, indicating that data
446 became neither more nor less stationary across blocks. All other RQA measures
447 reliably increased across the blocks of practice ($F(4, 56) = 5.11, p < .05$ for
448 determinism; $F(4, 56) = 75.36, p < .05$ for entropy; $F(4, 56) = 4.54, p < .05$ for
449 meanline, and $F(4, 56) = 2.71, p < .05$ for maxline). Just as for the fractal
450 measures, these differences occur specifically between block 3 and block 4.

451 Between blocks 3 and 4 difference contrasts revealed that determinism
452 increases, $F(1, 14) = 9.71, p < .01$, as does entropy $F(1, 14) = 10.77, p < .05$, the
453 average strength of attractor dynamics indicated by meanline $F(1, 14) = 7.90, p$
454 $< .05$, and strength of the strongest attractor indicated by maxline $F(1, 14) =$
455 $5.10, p < .05$. No other contrasts were statistically significant. However the
456 decrease in RQA measures was close to the threshold for statistical significance
457 for both entropy $F(1,14) = 4.0, p = .07$ and maxline $F(1,14) = 4.12, p = .06$. In
458 addition, a quadratic function gives a significant fit across blocks 3, 4, and 5, for
459 determinism $F(1, 14) = 5.25, p < .05$, entropy $F(1,14) = 6.13, p < .05$, and
460 maxline $F(1,14) = 7.79, p < .05$, and although meanline did not reach threshold
461 for significance it is close and in the right configuration. We did not anticipate
462 the overall downturn in RQA measures between blocks 4 and 5. The changing
463 RQA values are shown in Figs. 3a-3e.

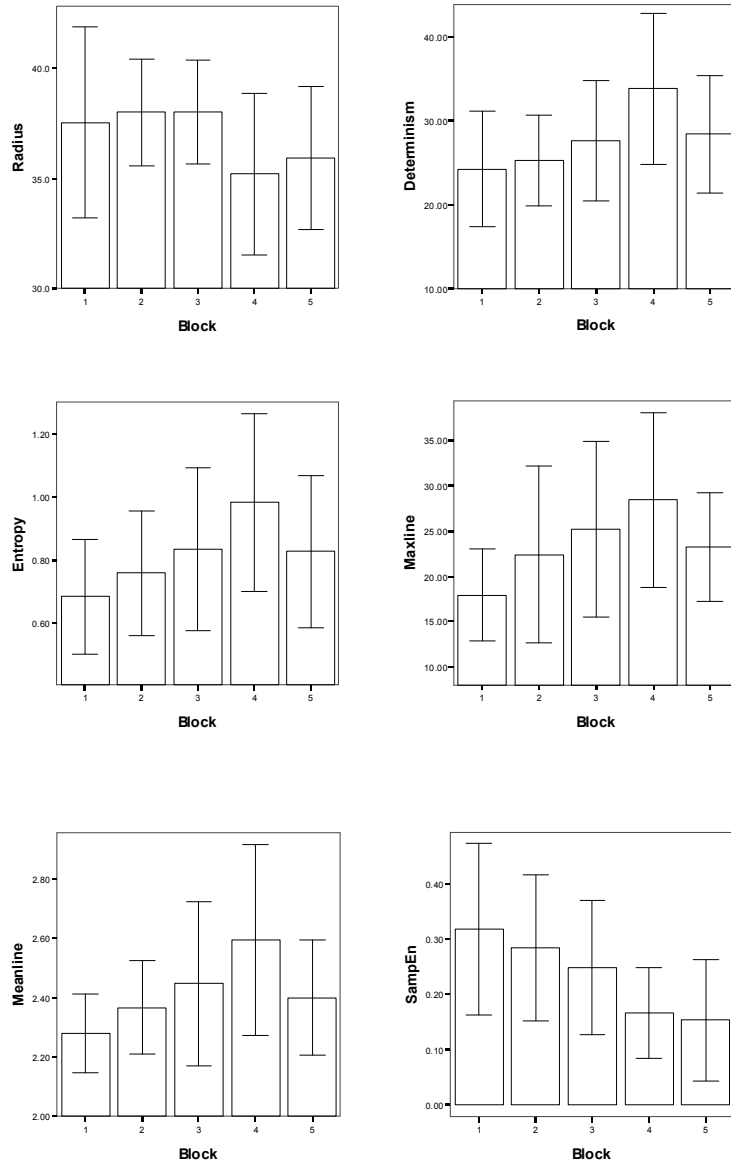
464 Most RQA measures change in the same direction across the first four
465 blocks of trials and then reverse direction in the fifth block. By comparison,
466 movement times decrease in the fourth block, and decrease even more in the
467 subsequent fifth block. These changes are not a function of a speed-accuracy
468 trade-off; the level of accuracy did not change. Perhaps the reversal of the global
469 pattern of change in the last block is due to fatigue. While we cannot know this
470 with certainty, it would contradict the idea that $1/f$ scaling itself is a fatigue
471 phenomenon (e.g. Wagenmakers et al., 2004), and is worth pursuing in future
472 work (with a sixth block for example), but we will not discuss this finding
473 further without a replication.

474 To investigate possible within-block changes, data series were divided
475 in four non-overlapping epochs of 256. RQA is a nonlinear tool, sensitive to
476 details of the full time series analyzed, and smaller epochs do not necessarily
477 combine to "equal" the outcome over an entire block. Within Block 1,
478 determinism, entropy, meanline and maxline dropped, and trend became less
479 negative: ($F(3,42) = 4.26, p < .05$; $F(3,42) = 5.12, p < .01$; $F(3,42) = 4.22, p$
480 $< .05$; $F(3,42) = 3.43, p < .05$; $F(3,42) = 6.57, p < .01$, respectively). The drop
481 occurred especially between epoch 1 and 2 (an apparent start up transient,
482 perhaps), the difference contrasts were $F(1,14) = 8.92, p < .05$; $F(1,14) = 12.16,$
483 $p < .01$; $F(1,14) = 4.22, p < .05$; $F(1,14) = 12.56, p < .01$; $F(1,14) = 7.39, p < .05,$
484 respectively. Otherwise, only one RQA parameter changed reliably; in block 3
485 trend changed to indicate that the data series became more stationary, $F(3,42) =$
486 $3.15, p < .05$.

487

Sample Entropy

488 The SampEn measures, like the RQA measures, effectively confirmed
489 the anticipated direction of change in dynamical degrees-of-freedom (see Fig.
490 3f). Over the five practice blocks, a repeated measures ANOVA revealed
491 decreasing SampEn, $F(4,56) = 3.87, p < .05$. Also a linear trend was observed
492 consistent with previous observations, $F(1,14) = 5.23, p < .05$. Within each
493 block, changes in SampEn were investigated by dividing the data series in
494 four non-overlapping epochs of 256 data points. However, no significant within-



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Fig. 3. Changes in (a) radius, (b) the percentage of determinism, (c) entropy, (d) meanline, (e) maxline and (f) sample entropy across blocks of practice.

502 block changes were observed. Also, none of the difference contrasts between
503 epochs were statistically significant in any of the practice blocks. Thus, SampEn
504 gradually decreased across, but not within blocks.

505

DISCUSSION

506 The primary finding of the present experiment is that movement time
507 variability shows more consistent time-dependent properties in more practiced
508 precision-aiming performance. Here, increasing skill with practice equals faster
509 movement times, both within and between training blocks, without trading-off
510 accuracy, plus increasingly clear $1/f$ scaling that also tracks the improving speed
511 of performance. Changes in $1/f$ scaling exponents (and other fractal statistics)
512 reliably track changes in the early phase of motor learning.

513 Our original prediction was thus confirmed. Practice better constrains
514 and coordinates interaction-dominant dynamics, to reduce degrees of freedom,
515 and so the structure of variation in movement times shows clearer signals of $1/f$
516 scaling. After practice movement dynamics became less random and more
517 patterned. In reconstructed phase space, the attractive region became more
518 deterministic and yielded a more complex structure (as indicated by higher
519 entropy). Other recurrence quantification (RQA) measures indicated increasing
520 system stability. And, after practice, a smaller radius captured the same
521 percentage of recurrent attractor states (see Fig. 3a), which, while not
522 statistically significant, replicates the pattern of the other variables, and suggests
523 that movement trajectories evolve in a more confined region through their
524 phase-space. Additional support for this claim comes from sample entropy
525 (SampEn), which drops with practice indicating a lower-dimensional
526 organization of coordinative structure. Thus practice adds constraints, which
527 make the task more feasible, or less difficult in a meaningful sense.

528 The difficulty of performing a motor task in a specific context generally
529 is often estimated by self-report or physiological measures. Alternatively, levels
530 of task difficulty are determined a priori based on reasonable assumptions about
531 difficulty that may or may not be true. We assumed for example that task
532 difficulty decreases with practice, and we then tracked practice effects using
533 linear and nonlinear tools in tandem, which revealed details of motor dynamics
534 that converge in a consistent story about practice effects. Namely, intrinsic
535 constraints acquired with practice change coordinative structures to reduce
536 degrees of freedom. If this is true, then the relative presence of $1/f$ scaling may
537 constitute a gauge for motor skill in closed motor tasks, and even difficulty or
538 workload in human performance more generally. The latter possibility would
539 conceive difficulty and workload as unsystematic perturbations on within-trial
540 motor coordination, and thereby random perturbations of $1/f$ scaling in repeated
541 measurements.

542 The presence of $1/f$ scaling, in general, contradicts any view of motor
543 coordination that regards variation in movement as uncorrelated noise imposed
544 on a motor signal. Thus, the presence of $1/f$ scaling poses challenges to many
545 conventional models of motor control (Torre, Delignières, & Lemoine, 2007).

546 Specifically, for the present data, Fitts' (1954) original model, and more recent
547 nonlinear models of precision aiming in the Fitts' task, have focused on central
548 tendency, not time-evolutionary properties (e.g. Mottet & Bootsma, 1999; Flach,
549 Guisinger, & Robison, 1996). The present results also contradict conjecture that
550 the relative strength of $1/f$ scaling increases with increases in task difficulty
551 (Chen, Ding, & Kelso, 2001; but cf. Van Orden et al., 2003) and the conjecture
552 that the effects of task difficulty or skill are discarded per se by focusing on
553 trial-by-trial variability (Wagenmakers et al., 2005).

554 In this regard, point to point movement times of each participant in
555 every block of trials of the present precision-aiming task fluctuated in the fractal
556 pattern of $1/f$ scaling. This outcome replicates previous wide-ranging
557 demonstrations that motor variability entails fractal $1/f$ scaling. Structure and
558 variation coexist in the time-evolutionary properties of motor behavior. This
559 outcome reinforces the crucial empirical analytic point that one must include
560 estimates of time-evolving structure of motor variability to derive an accurate
561 picture of motor behavior (Liu, Mayer-Kress, & Newell, 2006; Riley & Turvey,
562 2002; Slifkin & Newell, 1999; Treffner & Kelso, 1999).

563 All these outcomes support the perspective taken here that $1/f$ scaling in
564 motor (and cognitive) activity emerges from *interaction-dominant dynamics*.
565 Reciprocally interactive processes interlink across time scales to change each
566 other's dynamics and self-organize task performance (Van Orden et al., 2003). It
567 is known that $1/f$ scaling is most clearly seen in measurements when external
568 constraints are held constant, or changes are minimized (Gilden, 2001; Kello,
569 Anderson, Holden, & Van Orden, in press). These are the conditions of the
570 precision aiming task, which again reliably produced $1/f$ scaling. Yet under-
571 standing $1/f$ scaling as a reflection of self-organization is at odds with main-
572 stream psychological science. The central issue in that argument is the logical
573 possibility that $1/f$ scaling can appear as an exclusive consequence of ordinary
574 linear dynamics acting in a somewhat extraordinary fashion. As we explain next,
575 the outcome of the present experiment speaks to that argument as well.

576 Several independent sine waves plus random noise can be fitted to the
577 gross pattern of a $1/f$ signal (Granger, 1980; Pressing, 1999; Pressing & Jolley-
578 Rogers, 1997; Wagenmakers et al. 2004, 2005; Ward, 2002), as any pattern of
579 variation can be linearly modeled after the fact (Beran, 1994). However, such a
580 model must posit a special align parameter to integrate the independent
581 processes in the strict form of the scaling relation, or else must allow a primary
582 role for coincidence.

583 The present results further complicate such an account because they
584 demonstrate coordinated changes in the exact form of the scaling relation –
585 practice converges across blocks on clearer patterns of $1/f$ scaling. Scaling
586 exponents that estimate the overall structure of variation in movement times
587 change with practice in a systematic fashion. In the linear framework, scaling
588 exponents depend largely on the frequency and amplitude of variations in
589 specific component processes. Thus, to account for systematic change in the
590 exponent of $1/f$ scaling, linear models must add to their alignment parameter a

591 capacity to moderate or control components to change together, to insure that
592 their changes relative to each other maintain the $1/f$ relation between amplitude
593 and frequency.

594 This extra capacity of a controller-component would join other ad hoc
595 changes already implicated. For example, a linear model must introduce new
596 components each time a longer data set is collected (Van Orden et al., 2005),
597 and new components must be added when additional measurements are taken.
598 Additional measurements of the same repeated performance yield additional
599 uncorrelated streams of $1/f$ scaling (Kello et al., 2007; Kello et al., in press). In
600 other words, $1/f$ scaling behaves like we expect a fractal phenomenon to behave;
601 fractal time permeates collected data to their full extent. All these facts are
602 unexpected from linear models (Bak, 1996; Bassingthwaight, Liebovitch, &
603 West, 1994; Liebovitch & Todorov, 2000; Thornton & Gilden, 2005).

604 The interpretation of the presented results in terms of interaction-
605 dominant dynamics generates further insight into the nature of control and
606 coordination in perception and action. As constraints accrue with practice, new
607 lower-dimensional modes of intrinsic dynamics arise, which reduce the intrinsic
608 degrees-of-freedom, Scaling exponents move closer to the -1 scaling exponent
609 of hypothetical $1/f$ scaling because practice is a means to add constraints in
610 behavior and reduce degrees of freedom for behavior, and thereby reduce
611 across-trial and within-trial sources of random variation in measures of behavior.

612 Skilled and unskilled movements emerge to satisfy the constraints,
613 extrinsic and intrinsic, of the task at hand. Movements are not solutions to a
614 mechanical equation. Significant changes in $1/f$ scaling for identical task
615 conditions suggest dynamics modulated by the coupling of task and participant,
616 not just by properties tasks. Parallel changes between fractal, complexity, and
617 traditional performance measures motivate this claim and previous findings also
618 support this conclusion (Pressing & Jolley-Rogers, 1997). Thus fractal dynamics
619 are informative about task complexity, but complexity must take into account
620 both task and participant.

621 This brings us to a final question. Why $1/f$ scaling? Why do added
622 constraints, that better coordinate the dynamics of brain and body with the
623 dynamics of task requirements, yield scaling exponents closer to the ideal form
624 of $1/f$ scaling? $1/f$ scaling is the idealized pattern of interaction-dominant
625 dynamics that separates chaotic variation from rigid order. $1/f$ scaling is also the
626 idealized pattern of interaction-dominant dynamics that never strays far from
627 choice points, or critical points. This insures flexibility to adjust kinematics even
628 as behavior is realized and even to produce entirely novel kinematics when
629 necessary.

630 Flexibility also equals vulnerability with respect to inevitable and
631 ubiquitous perturbations of measured behavior, of all sorts. Such perturbations
632 contribute random variation, which will whiten the signal of $1/f$ scaling.
633 Interaction-dominant dynamics perturbed to be less near critical points and more
634 toward chaotic dynamics will appear empirically as a whitened $1/f$ signal. If
635 these hypotheses are significant, then $1/f$ scaling-exponent will soon be widely

636 recognized as an index or order parameter of coordination in human
637 performance.

638

ACKNOWLEDGMENT

639

640 We acknowledge funding support from NSF BCS-0642718 and NSF
641 DHB-0728743, Guy Van Orden, Principal Investigator. Maarten L. Wijnants
642 wrote this article while he was affiliated with the Department of Psychology,
University of Cincinnati, Ohio.

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